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Optimal fines in libraries and documentation centers

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Abstract

In this paper we try to determine the optimal level of the fines imposed on library users. These fines are levied to deter people from keeping books too long or even damage or steal them. The fines should be high enough to prevent abuses but not excessively high. Our model will be based on the work of Polinsky and Shavell on optimal fines for polluters. It will be proved that the optimal fine should depend on the level of harm done, the probability of detection and the variable enforcement cost. We will illustrate this with an example.

1. Introduction

Library users who do not return their books on time, or sometimes do not return them at all, are a constant nuisance to the librarian. To this list of anti-social behavior we can also add the damaging of books by sand, food, ripping pages or adding – irrelevant – remarks.

This explains why each library has a set of rules to explicitly prohibit such abuses. Prohibiting is one thing, but to have the desired effect, the librarian needs to be able to impose a sanction. This penalty can consist of temporary or permanent exclusion, fines or, when it concerns a serious crime (e.g. theft of rare books), the police can be called for. In this paper we will concentrate on using fines as a penalty instrument.

We will now try to induce the correct level of the fine. Indeed, to levy too low a fine would be a jest that costs more than it yields. Nevertheless draconian punishments cannot be justified either in a (public) library.

Therefore we will develop a model to optimize these fines. Our research is an adaptation of the economic model of Polinsky and Shavell (1992) that tries to determine the optimal fine for violations of environmental law. We will show that the fine is closely linked with the effort made by the library to deter the violators (the monitoring effort).

For more traditional discussions of the fining problem we refer the reader to the books of Burgin & Hansel (1984) and Martin & Park (1998).

2. General principles

Preventing and discovering theft, overdues and damages induces costs to the library. These costs are: installing a security system, following up files (for which you might need special software), sampling books or users, which takes time to the library personnel. It is clear that the optimal level of a fine depends on the costs one has to incur. The optimal level of enforcement, here the optimal level of detection, is also dependent on these costs.

We can make a distinction between two kinds of costs: fixed and variable enforcement costs. Fixed costs do not depend on the number of offenders. Security system installation costs are an example of fixed costs. Variable enforcement costs do depend on the number of violators. Examples are: stamps needed to send reminder cards, or telephone calls made to people who have not returned their books.

We will show that the optimal fine should depend of the level of harm done, the probability of detection and the variable enforcement costs.

Even if it were possible, it is too expensive to catch and fine every offender. A certain level of tolerance is always recommended.

3. The Polinsky-Shavell model

In this model we assume that each library user is a potential offender. Indeed, being able to keep a book for as long as you like or clipping illustrations from an encyclopaedia entails benefits for the person who can do this. Of course, in reality, there will always be library patrons who will adhere to a library's regulation. Offenders have a certain probability of getting caught. Detection brings along costs for the library.

3.1 Notation

We will first introduce the notation and will next give further explanations concerning the assumptions of this model.

The following symbols will be used:

H: stochastic variable, defined over all library users, giving the damage induced by violating a library's regulations g(h) : continuous density function of H, defined on $[0,\infty[$ and with distribution function G. Hence:

$$P(H \leq t) = G(t) = \int_{0}^{t} g(h)\,dh$$

B: stochastic variable, denoting the benefit for the offender when contravening the regulations

r(b) : continuous density function of B defined on $[0,\infty[$; its distribution function is denoted as R

c : fixed costs , $c \geq 0$

p(c) : probability of catching an offender, given the level of fixed costs (c); p is a concave, increasing function of c, starting at zero, so $p(0) = 0$, $p'(c) > 0$, $p''(c) < 0$

W: stochastic variable denoting the wealth (financial means) of an individual (library user)

w : financial means of a particular library user

f : penalty function; f(h) is the fine if the damage is h, $0 \leq f(h) \leq w$

k : variable cost for imposing fines

3.2 Assumptions of the P-S model

The assumptions underlying the P-S model are the following.

Users are risk neutral. This means that they need not be compensated for incurring risks. They just look at the expected value of the action, not at its variation.

The damage is an observable so that the fine can be an increasing function of the damage level.

Benefit, b, cannot be observed.

The stochastic variables H and B are independent with known density functions. Note that this is a fairly restrictive requirement. Usually a higher benefit goes together with a greater damage.

The probability of catching an offender follows, as a function of the fixed costs, the law of diminishing returns. Hence it is a concave function of c.

The imposed fine can not be larger than the financial means, w, of the user. Indeed, it can never be justified that a library fine leads to a person's loss of all private means.

The probability of detection is the same at all damage levels. Detecting the theft of a cheap novel has an equal likelihood as detecting the theft of a science book.

Any user will perform a forbidden deed if his/her expected benefit is larger than the expected fine, i.e. p times f. In economic terms this means that individuals act rationally:

$$b \geq pf \tag{1}$$

The social welfare, expressed in monetary terms and denoted as SW, is, for fixed c, equal to the double sum (actually integral) over all benefits and all damage levels of all individual benefits obtained by forbidden deeds, diminished by the damage caused and the expected enforcement costs.

Hence:
$$SW = \{ \int_0^\infty (\int_{pf}^\infty (b - h - pk) r(b) db) \, g(h) dh \} - c \tag{2}$$

We see that expression (2) takes into account the fact that the cost to determine the probability of detection is independent of the number of offenders. The expected cost for raising fines increases with an amount equal to pk for every user violating the library regulation.

3.3 *Goal*

The goal of the library manager is to maximize the cultural and intellectual environment (including recreational aspects) of the clients. This is what we mean here by the term social welfare (SW). However, every book, video cassette or multimedia package that helps to reach this goal represents (also) a monetary value. Hence, taking an economic point of view we will work in terms of maximizing the monetary value of SW. The library manager can attain this goal by optimizing the fixed cost c (and thus, implicitly, the probability of catching an offender, p, which depends on c) and the penalty function f(h). The higher the fixed costs for prevention and detection, the higher the probability that an offender will be caught. This is illustrated in the following figures (Fig.1,2).

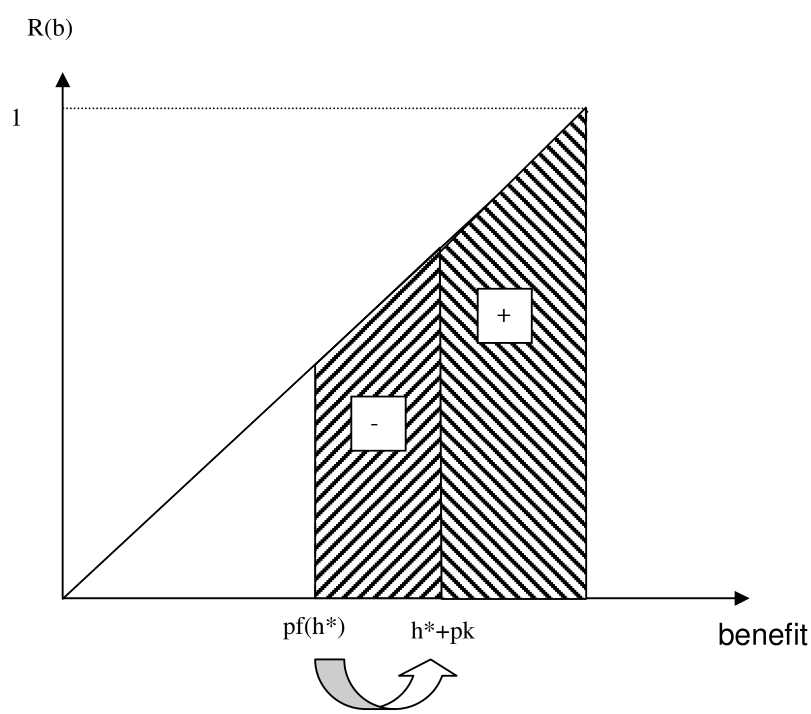

Fig.1 Determination of the optimal fine in the case of constant damage h*

The library manager's goal is to maximize social welfare. We assume, in this example, that the benefit b is uniformly distributed over a certain interval. Hence, its distribution function is an increasing line (over this interval), beginning in zero and ending at one. The social damage done by a certain unlawful act is h* + pk. This is: the actual damage plus the expected variable enforcement costs.

A (rational) library user will perform a forbidden deed only if the benefit is larger than the expected fine. This means that we have to determine the optimal fine in such a way that it is equal to the social damage. Only then potential offenders will take the real costs into account and take those decisions that maximize social welfare. (We assume here that personal wealth is not relevant, as it usually is the case). Figure 1 illustrates the case that the expected fine is smaller than the social costs. Too many violations will occur and the area of the region indicated with the **-** sign is a measure for the burden on the library. Optimizing (here raising) the fine can solve the problem (recall that h* + pk denotes the social damage done by an unlawful deed). This action is symbolized by the arrow.

Figure 2 illustrates a similar situation, but now we hold the benefit, b, constant. We have taken an arbitrary, but realistic, damage function (again taking diminishing returns into account). As the fine is a linear function of the damage, this means that it has the same shape. In Fig.2 the expected fine is larger than the social costs. Such a fine leads to a general welfare that is smaller than the optimal level. Reducing the expected fine to a level equal to the social costs increases general welfare. Again this action is symbolized by an arrow.

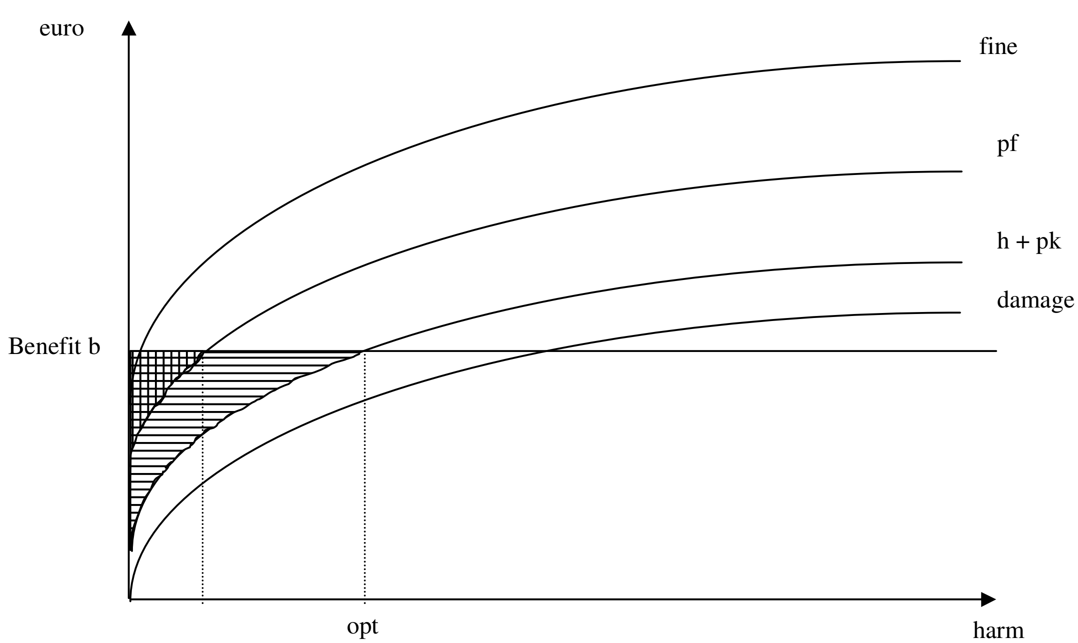

Fig.2 Determination of the optimal fine in case of constant benefit

4. An analysis of the P-S model

4.1 Determination of the optimal fine

The problem we want to solve is to find, for every damage h, the fine f that maximizes the following expression:

$$\int_{pf}^{\infty} (b - h - pk)\, r(b)\, db \qquad (3)$$

Expression (3) is the social welfare at a fixed damage level h. In order to find its maximum we take the derivative of (3) with respect to f. This yields:

$$-p.(pf-h-pk).r(pf) \qquad (4)$$

We note that (4) is positive for $pf < h + pk$, hence for $f < h/p + k$; it is zero for $pf = h + pk$, i.e. for $f = h/p + k$; and negative for $pf > h + pk$, i.e. for $f > h/p + k$.

This leads to the following conclusion:

The optimal fine, denoted as $f°$, is:

$$f°(h) = h/p + k\,, \quad \text{if } h/p + k \leq w, \text{ i.e. if } h \leq p(w-k) \qquad (5)$$

$$f°(h) = w \qquad \text{otherwise.}$$

4.2 Discussion

- If $h \leq p(w-k)$ (5) and $w \geq k$

  then the optimal fine, for a damage h, is $f°(h) = h/p + k$

  In this situation, the expected fine is:

  $$E(f) = pf = h + pk \qquad (6)$$

  We see that the expected fine (for the offender) is equal to the expected cost to the library. Hence, in this situation the deterrence effect of fines is optimal. We could say that the level of non-compliance to the library regulations is socially optimal.

  If $w < k$ (say for a poor citizen) then h never satisfies inequality (5) and: $f°(h) = w$ Of course, if someone is so poor that his/her financial means are smaller than the variable costs associated with the detection of a violation of a library regulation, it becomes socially unacceptable to impose a fine on this person.

- If $h > p(w-k)$,

  then the optimal fine is equal to the financial means of the offender:

  $$f°(h) = w \qquad (7)$$

  and the expected fine becomes:

  $$E(f) = pw \qquad (8)$$

  The expected fine is too small (because p is too low), namely smaller than the expected social cost. This means that the deterrence effect is too low.

Probably this library will suffer from too many unreturned books, too many damages to books and too many thefts.

4.3 Determination of the optimal probability of catching an offender

Social welfare (SW) can be expressed as:

$$\begin{aligned} \mathrm{SW} = & \int_0^{p(w-k)} \Big(\int_{h+pk}^{\infty} (b-h-pk)r(b)db \Big)\, g(h)dh \\ & + \int_{p(w-k)}^{\infty} \Big(\int_{pw}^{\infty} (b-h-pk)r(b)db \Big)\, g(h)dh - c \end{aligned} \tag{9}$$

The first integral sums over small damages, the second one over the larger ones. Note that if h < p(w-k) then pf = h + pk, while for h > p(w-k), pf = pw. So the second integral takes the financial means of offenders into account. Taking the derivative of (9) with respect to c and setting the result equal to zero, yields (for a proof see (Rousseau, 1997)) :

$$\begin{aligned} & r(p(c)w)p'(c)w \int_{p(w-k)}^{\infty} (h + p(c)k - p(c)w) g(h)dh \\ & = 1 + p'(c)\,k \left\{ \int_0^{p(c)(w-k)} \big[1 - R(h+p(c)k)\big] g(h)\; dh + \big[1 - R(p(c)w)\big]\big[1 - G(p(c)(w-k))\big] \right\} \end{aligned} \tag{10}$$

where R and G denote the distribution functions associated with the density functions r and g. Expression (10) implicitly determines the optimal p°. We assume that all other mathematical conditions are satisfied so that we have a maximum (and not a minimum).

4.4 Discussion

Expression (10) is the first order condition of the optimization problem. The left hand side of (10) is equal to the marginal benefit of raising c and therefore p. Looking at the integral, we see that only the high damages, h > p(w-k), are considered. This indicates that we are in the case of underdeterrence or the expected fine pf = pw is smaller than the social cost h + pk. Recall that an individual will damage (or steal) a book if the benefit of doing so is higher than the expected fine, or b > pf. In this case individuals will act wrongly if b > pw. To obtain the social optimum, only the individuals with b > h + pk should damage the books. Raising the probability p to p_N now only deters those library users that would obtain a benefit pw of the abuse. The individuals with a social cost (h + pk) larger than pw and smaller than p_Nw, will now act optimally and will not damage books. Hence (for each such individual) the welfare increases with an amount of h + pk - pw. This amount is equal to the previous loss of underdeterrence for these individuals.

The right hand side is the marginal cost of raising c and hence p. It has two components: the direct cost of raising the fixed costs c (an increase with one unit costs you one unit) and the indirect effect of the increase in variable costs because of more frequent fines. The total variable costs now increase with $p'(c).k$ corrected for the influence of p (or c) on the optimal fine and the social costs.

4.5 *How does the optimal inspection probability $p°$ depend on fixed and variable enforcement costs?*

* The influence of variable enforcement costs k

We consider the behavior of p for constant fixed enforcement costs c. If $k = 0$, p takes some value between zero and one and is only dependent on c. On the other hand, if k tends to infinity, p tends to zero. This means that in reality it is better not to enforce the regulation.

It is not clear how p will behave between these two extremes: it may increase or decrease. The direction in which p will move depends on the difference between the marginal benefit and the marginal costs. For increasing k the marginal benefit of increasing p will increase too because social loss by underdeterrence decreases. However, it is not possible to determine the direction in which the marginal costs will change. On the one hand there is an increasing tendency as the variable enforcement costs will increase when more people get caught, on the other hand there is also a decreasing tendency as less people will offend the rules with increasing k which leads to a decrease in variable costs.

We note that the reason why the optimal probability to catch offenders $p°$ must tend to zero is that otherwise the total variable costs would tend to infinity with increasing costs, k, and a positive fraction of people getting caught.

* The influence of fixed enforcement costs

We first introduce some extra notation:

λ : the productivity of costs c, $\lambda > 0$

$p(\lambda c)$: the probability of catching an offender (depending among other things on the sampling frequency)

So, the higher λ, the higher p for a fixed value of c.

If λ tends to zero, $p°$ also tends to zero. Further, if λ increases, the behavior of $p°$ is not clear: it can increase or decrease. If, however, λ becomes very large, then $p°$ tends to $\hat{p}$. This is, the optimal p if it were possible to increase the probability of catching an offender without increasing the costs. Note that $\hat{p}$ can be strictly smaller than one.

The fact that the marginal costs of increasing the probability of catching an offender can be larger than the benefits accrued from deterring library users explains why $p°$ is zero for small values of λ.

The reason why $\hat{p}$ is not necessarily equal to one and, indeed, is usually strictly smaller than one is that even if there were no costs associated with fining people there still are increasing variable costs when the number of people caught increases.

5. An example

We will consider two cases: the theft of a book and the damaging of a book.

First we need to estimate the probability (p) of detecting the violation. If you steal the book, you can only get caught in the library itself. Once you get out of the building, we suppose you are safe. Many libraries do not allow carrier bags to be taken inside, making detection of theft easier. Still, if no magnetic detection equipment is installed, the probability of detection will be very low. Arbitrary we will set p = 20 percent. We will also consider the case that magnetic detection equipment is installed: in that case we put p = 95%. In the case of damage to a book, the probability of detection will again be low. Only when the damage is discovered immediately, the wrongdoer can get caught. Therefore, and for simplicity, we will use the same p = 20% here.

Next, we need to estimate the harm done (h). In case of theft, we assume that the damage is equal to the cost of buying a new book. We assume that the market value reflects all the relevant characteristics of the book. Here we will assume that it costs the library 25 euro to buy the book again, so h_T = 25 euro. If the book is damaged, we assume that the library will have to buy the book sooner than scheduled, say four years earlier. We will equal the harm done to the difference between the present value of the market price (say 20 euro) in four years and the price of the book now. This gives, with a discount rate of 4%,

$$h_D = 25 - \frac{20}{(1+0.04)^4} = 25 - 17.1 = 7.9 \ euro$$

or with a discount rate of 2% and the same market price

$$h_D = 25 - \frac{20}{(1+0.02)^4} = 25 - 18.5 = 6.5 \ euro$$

or when the book is bought two years earlier

$$h_D = 25 - \frac{20}{(1+0.02)^2} = 25 - 19.2 = 5.8 \ euro$$

Finally we also need an estimate of the variable monitoring and enforcement cost (k). We assume that 2 letters are written and 2 phone calls are made, or k = 2*0.43 + 2*0.75 = 2.36 euro.

Now we can calculate the optimal fine in our two cases (using formula (5)):

$$f_T = \frac{h_T}{p_T} + k = \frac{25}{0.2} + 2.36 = 127.36 \ euro$$ and

$$f_D = \frac{h_D}{p_D} + k = \frac{7.9}{0.2} + 2.36 = 41.86 \ euro$$ or

$$f_D = \frac{h_D}{p_D} + k = \frac{6.5}{0.2} + 2.36 = 34.86 \ euro$$ or

$$f_D = \frac{h_D}{p_D} + k = \frac{5.8}{0.2} + 2.36 = 31.36\,euro$$

In case detection equipment is installed (i.e. taking p = 0.95) these fines becomes respectively: 28.68, 10.7, 9.2 and 8.47 euro. This illustrates the philosophy that if the probability of detection is small the fine must be larger in order to have the same deterrence effect.

6. Conclusion

The general structure to determine fines has been determined. Yet, the optimal probability to catch offenders can only be determined if one, moreover, has information concerning enforcement costs and the costs of fining people. The formula given by this model has some intuitive plausibility. Offenders must pay the damage and the costs to catch and fine them. We may say that the basic formulae of the Polinsky-Shavell model are easy to apply. The only thing the librarian must do is to determine the fine he/she would normally find to be adequate. Then this value must be augmented by the costs for detection and enforcement. This implies that each library or documentation center can determine its own fines. No external data are necessary.

Note that this article is a typical example of traditional bibliometrics, namely the application of a mathematical model in a library setting. It constitutes, moreover, another link between the fields of informetrics and economics (cf. Rousseau, 1994).

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Appendix (not published)

Finding the optimal probability of detection, derivation of expression (10).

Social welfare SW is maximized with respect to c.

$$\mathrm{SW} = \int_{0}^{p(c)(w-k)} \int_{h+p(c)k}^{\infty} (b-h-p(c)k)\, r(b)\, db)\, g(h)\, dh$$

$$+ \int_{p(c)(w-k)}^{\infty} \int_{p(c)w}^{\infty} (b-h-p(c)k)\, r(b)\, db)\, g(h)\, dh - c$$

Taking the derivative with respect to c gives: $\frac{\partial W}{\partial c} = \frac{\partial I_1}{\partial c} + \frac{\partial I_2}{\partial c}$:

The first integral yields:

$$\frac{\partial I_1}{\partial c} = \int_{0}^{p(c)(w-k)} \frac{\partial}{\partial c}\left[\int_{h+p(c)k}^{\infty} (b-h-p(c)k)\, r(b)\, db\, g(h) \right] dh$$

$$+ \int_{p(c)w-p(c)k+p(c)k}^{\infty} (b-p(c)w+p(c)k-p(c)k)\, r(b)\, db \;\; g\big(p(c)(w-k)\big) \;\; p'(c)(w-k)$$

$$= \int_{0}^{p(c)(w-k)} \left[-\underbrace{(h+p(c)k-h-p(c)k)}_{=0} r(*)\, g(h) + \int_{h+p(c)k}^{\infty} -p'(c)k\, r(b)\, db \right] g(h)\, dh$$

$$+ \int_{p(c)w}^{\infty} (b-p(c)w)\, r(b)\, db - g\big(p(c)(w-k)\big) p'(c)(w-k)$$

The second one gives:

$$\frac{\partial I_2}{\partial c} = \int_{p(c)(w-k)}^{\infty} \frac{\partial}{\partial c}\left[\int_{p(c)w}^{\infty} (b-h-p(c)k)\, r(b)\, db \;\; g(h) \right] dh$$

$$- \int_{p(c)w}^{\infty} \Big[\big(b-p(c)w+p(c)k-p(c)k\big)\, r(b)\, db \;\; g\big(p(c)(w-k)\big) \;\; p'(c)(w-k) \Big] - 1$$

$$= \int_{p(c)(w-k)}^{\infty} \left[\int_{p(c)w}^{\infty} -p'(c)k\, r(b)\, db \;\; g(h) - (p(c)w-h-p(c)k)\, r\big(p(c)w\big) \;\; g(h)\, p'(c)w \right] dh$$

$$- \int_{p(c)w}^{\infty} \Big[(b-p(c)w)\, r(b)\, db \;\; g\big(p(c)(w-k)\big)\, p'(c)(w-k)) \Big] - 1$$

$$= -\int_{p(w-k)}^{\infty} g(h)\,dh \int_{pw}^{\infty} r(b)\,db \quad p'(c)k + r(pw)\,p'(c)\,w \int_{p(w-k)}^{\infty} (h+pk-pw)\,g(h)\,dh$$

$$-\int_{pw}^{\infty} \left[(b-pw)\,r(b)\,db \;\; g\big(p(w-k)\big)\,p'(c)(w-k))\right] - 1$$

Adding these two results gives:

$$\frac{\partial W}{\partial c} = \frac{\partial I_1}{\partial c} + \frac{\partial I_2}{\partial c}$$

$$= \int_{0}^{p(w-k)} p'(c)\,k\,g(h) \left[\int_{h+pk}^{\infty} r(b)\,db\right] dh + \int_{pw}^{\infty} (b-pw)\,r(b)\,db - g\big(p(w-k)\big)\,p'(c)(w-k)$$

$$-\int_{p(w-k)}^{\infty} g(h)\,dh \int_{pw}^{\infty} r(b)\,db \quad p'(c)k + r(pw)\,p'(c)\,w \int_{p(w-k)}^{\infty} (h+pk-pw)\,g(h)\,dh$$

$$-\int_{pw}^{\infty} \left[(b-pw)\,r(b)\,db \;\; g\big(p(w-k)\big)\,p'(c)(w-k))\right] - 1$$

$$= p'(c)\,k \int_{0}^{p(w-k)} g(h)\left[1 - R(h+pk)\right] dh + r(pw)\,p'(c)\,w \int_{p(w-k)}^{\infty} (h+pk-pw)\,g(h)\,dh$$

$$-\,p'(c)\,k\left[1 - (p(w-k))\right]\left[1 - R(pw)\right] - 1$$

Putting: $\dfrac{\partial W}{\partial c} = \dfrac{\partial I_1}{\partial c} + \dfrac{\partial I_2}{\partial c} = 0$

yields:

$$r(pw)\,p'(c)\,w \int_{p(w-k)}^{\infty} (h+pk-pw)\,g(h)\,dh =$$

$$1 + p'(c)\,k\left\{\int \left[1 - R(h+pk)\right] g(h)\,dh + \left[1 - G(p(w-k))\right]\left[1 - R(pw)\right]\right\}$$

QED.