LIMBURGS
UNIVERSITAIR
CENTRUM

Instituut voor Materiaalonderzoek

Faculteit Wetenschappen

Statistical techniques for planning type I singly censored reliability experiments with two stress factors

Proefschrift voorgelegd tot het behalen van de graad van

Doctor in de Wetenschappen

aan het Limburgs Universitair Centrum, te verdedigen door

KRISTOF CROES

Promotor : Prof. dr. L. De Schepper
Co-promotoren : Prof. dr. G. Molenberghs
Prof. dr. J. Beirlant

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Een woord van dank...

Gezien dit doctoraat een brug tracht te slaan tussen de wereld van de statistiek en die van het onderzoek naar de betrouwbaarheid van IC's, zal de lijst met te bedanken personen onvermijdelijk zeer lang zijn. Vele mensen hebben mij gedurende dit werk geholpen en ongetwijfeld zal onderstaand lijstje niet helemaal volledig zijn...

Vooreerst zou ik Prof. Dr. Lambert Stals willen bedanken voor de geboden mogelijkheid om mijn werk op het IMO te kunnen uitvoeren.

Mijn promotor Prof. Dr. Luc De Schepper zou ik willen danken voor het aanreiken van het zeer boeiende onderwerp en om mij in te leiden in de wereld van het onderzoek naar de betrouwbaarheid van elektronische componenten. Het overzicht dat hij over de dingen hield was voor mij altijd weer verhelderend.

Mijn copromotor Prof. Dr. Geert Molenberghs heeft mijn werk van begin tot eind met zeer veel interesse gevolgd, ondanks het feit dat voor hem al die termen uit de fysica niet altijd voor de hand liggend waren.

Ook voor mijn andere copromotor, Prof. Dr. Jan Beirlant, zal het zeker een hele klus geweest zijn om zich in te leven in een totaal ander onderzoeksdomein.

Dr. Ward De Ceuninck zou ik willen bedanken voor zijn onuitputtelijke bron van goede ideeën en suggesties. Ik ben blij te weten dat ik hem "warm" heb gemaakt voor statistiek.

De overige leden van mijn doctoraatscommissie, Prof. Dr. Gilbert Knuyt, Prof. Dr. ir. Herman Maes en Dr. ir. Guido Groeseneken, zou ik willen danken voor de getoonde interesse doorheen deze vier jaren.

Het IWT bedank ik voor het toekennen van een specialisatiebeurs waardoor ik financieel niets tekort kwam gedurende de duur van het doctoraat.

Dr. Luc Tielemans heeft aan dit doctoraat een iets meer "industriële" tint gegeven. Wetenschap komt voor hem "uit de buik". Maar goed dat dat buikje niet te dun is...

Ik wens ook al mijn andere IMO-collega's te danken voor de fijne samenwerking doorheen deze vier jaar. In het bijzondere dank ik Dr. Jean Manca omdat hij mij er constant heeft toe aangezet om mijn werk te publiceren. Erik Thoonen en ook Rainer Petersen dank ik voor de vele "onderhoudswerken" aan mijn PC.

Paul Meyvisch, Liesbeth Bruckers en Philippe Roussel zou ik willen danken voor het aandachtig nalezen van dit werk. Vooral de jarenlange ervaring en het opgebrachte enthousiasme van Philippe hebben mij vertrouwen gegeven.

Twee mensen hebben "vanaf de start" achter mij gestaan, in alles wat ik deed. Gedurende mijn ganse studietijd hebben zij met mij meegeleefd. Ma en pa: bedankt voor alles...

Mijn vriendenkring en de mensen uit mijn familie mogen aan dit lijstje zeker niet ontbreken. Een mens is een sociaal voelend wezen en die sociale contacten heb ik altijd op prijs gesteld.

Aan het eind van dit lijstje komt er nog iemand die ik eigenlijk helemaal vooraan, in het vet gedrukt, zou moeten zetten. Men spreekt soms over de vrouw achter de man. Nu, als je onze twee posturen vergelijkt lijkt het me zeker niet overbodig om ook eens de vrouw vóór de man te plaatsen. Krista heeft me altijd gesteund en ik dank haar daar dan ook oprecht voor...

Bedankt allemaal,

Kristof

Contents

Preface

A crucial consequence of the ongoing miniaturization in the microtechnology is that producing reliable integrated circuits (IC's) gets more and more difficult. The subject of this thesis can be situated in the world of testing the reliability of such IC's. These tests are important because the competitiveness of manufacturers strongly depends on the demonstrable reliability of the produced IC's.

The major building blocks of an IC can be subdivided into off-chip and on-chip elements. At the on-chip level, active components as well as so-called back-end elements have to be considered. Active components, mainly transistors and diodes, care for the information contained in an IC. Back-end elements can be defined as elements taking care for the interconnection between the active components. This work concerns reliability tests on on-chip elements.

One of the major failure mechanisms of on-chip interconnects is electromigration [SCO91]. The basic principle of electromigration is known: as a consequence of an electrical current, atoms in a conductor migrate in the direction of the electrons. The major failure mechanism of a transistor is hot-carrier degradation [VUI98], while capacitors mainly age due to time-dependent dielectric breakdown [MAR98].

One of the crucial problems when testing the reliability of new components is that their lifetime under normal operating conditions is always extremely long (in the order of years). For that reason, the physical mechanisms that are responsible for a component to fail are studied and methods are established for accelerating these mechanisms. The electromigration mechanism, for example, can be accelerated by subjecting the interconnects to elevated temperatures and current densities. Then, the failure times of the test components operating under these accelerated conditions are measured and models are developed for extrapolating these data to real life conditions. The final step of a reliability experiment is the statistical analysis of the results.

The problem this work deals with can best be defined as: "How to plan a reliability experiment so that the total number of available test components and the total amount of available measurement time can be optimally used for predicting the reliability of the components under consideration?". In this thesis, restriction will be made for reliability experiments with two stress factors.

In the first chapter of this work, a general introduction of all aspects of a reliability experiment will be given. Both the physical aspect of how IC's age as well as the statistical aspect of analyzing reliability data sets will be introduced.

In chapter 2, the mathematical technique that has been developed for planning experiments that allow an accurate reliability estimation will be described. This technique is a merger, an extension and an improvement of two techniques proposed in the literature. For that reason, a discussion of the literature will be given as well.

Because the technique explained in chapter 2 has some less desirable practical and mathematical properties, methods are proposed to account for them. A discussion of these methods will be presented in chapter 3.

Concerning the assumptions made by the technique discussed in chapters 2 and 3, two important remarks have to be made. These remarks are discussed in chapter 4

In the final chapter of this thesis, chapter 5, the major conclusions of this work will be drawn.

Important to add is that appendix A summarizes the *terms and abbreviations* and that appendix B summarizes the *notation* used in this text.

1. Introduction

Today, integrated circuits (also called IC's or chips), best defined as incredibly complex modules that store computer memory or provide logic circuitry, are indispensable in a human life. IC's are made for personal computers, automobiles, home appliances, telephones, etc.

The subject of this thesis is situated in the world of testing the reliability of IC's. The aim of this work is to develop techniques for planning experiments that allow the accurate determination of an IC's lifetime, or of one of its elements.

In this chapter, different aspects concerning the assessment of the reliability of IC's are be introduced. In section 1.1, the structure of an IC is discussed in general. Both the importance and the major difficulties of producing reliable IC's are emphasized. Section 1.2 is devoted to the introduction of a reliability experiment. Here, the terms "accelerated test", "stress factor" and "lifetime model" are defined. Section 1.3 deals with the statistical analysis of reliability experiments. In the final section of this chapter, section 1.4, the problem that this thesis wants to account for is defined.

1.1. Reliability in microelectronics

In this section, the general structure of an IC is described and the importance of producing reliable IC's is emphasized. This section also deals with the main causes of failure of some typical components of an IC.

1.1.1. The general structure of an IC

The basis of an IC consists of a silicon (Si) wafer. First, this wafer is cut to size. Next, mainly two types of elements are added: active and passive components. Active components are mainly transistors and diodes. Passive components can be capacitors or on-chip interconnects. Capacitors are used in order to store charges in an IC. On-chip interconnects are needed for the interconnection between active components. Other commonly used names for on-chip interconnects are metal stripes or simply lines. Simplified, one can state that active components care for the information contained in an IC, while the task of interconnects is to transport this information through the IC.

1.1.2. Miniaturization and its consequence with respect to reliability

In order to obtain high-volume, low-cost electronic packaging technologies which are necessary for manufacturers to produce more sophisticated consumer products like camcorders, cameras and digital data books, it is needed to produce IC's which are as small as possible. The increasing miniaturization of IC's does not only require an increasing number of active components per unit of volume, but also that the on-chip interconnects get more narrow. In the seventies, interconnect widths of about 3 μm were common, while nowadays, it is possible to produce interconnects as narrow as 0.2 μm. Interconnects narrower than 1 μm are called sub-micron interconnects.

With respect to reliability, the miniaturization of IC's has two consequences. First, the reliability requirements of the active and passive components are much higher simply because the number of these elements in an IC increases. Second, physical mechanisms which did not play a role in the aging process of older technologies can get important in new technologies. These two reasons make it more and more difficult to produce IC's that meet the reliability requirements.

1.1.3. Importance of reliability and today's reliability requirements

For most manufacturers, competitiveness is the most important motive to produce reliable IC's. The competitiveness between manufacturers strongly depends on the demonstrable reliability of the produced IC's. Costs difficult to measure for producers of IC's are the intangible costs related to the loss of customer confidence caused by a failed product. Safety is another, maybe less widespread, motive to produce reliable IC's. Manufacturers producing IC's for manned spacecraft's or for ABS-systems in cars, for example, have to guarantee very high reliability due to safety reasons.

Requirements used to accept or reject a new technology are often based on the so-called x%-percentile, defined in appendix A. One such requirement is that the 0.005% percentile is higher than 1 year. Another reliability requirement is that, within a period of 20 year, no more than 0.01% of the total population can have failed.

Reliability requirements can also be based on so-called FIT-rates, defined in appendix A. Today, FIT-rates in the order of 10 FIT are demanded.

1.1.4. Main causes of failure of some typical components of an IC

In this section, the main causes of failure of some typical components of an IC are described. The following causes are considered: the hot-carrier degradation of MOSFET's, the time dependent dielectric breakdown of thin dielectrics, the electrostatic discharge behavior of IC's and both the electromigration and stress migration phenomenon in interconnects.

1.1.4.1. Hot-carrier degradation of MOSFET's:

Hot-carrier degradation is the most dominant failure mechanism of Metal Oxide Semiconductor Field Effect Transistors (MOSFET's). A MOSFET is an active component and it is one of the most elementary building blocks in the design of an IC. A typical n-type MOSFET consists of a source and a drain, two highly conducting n-type Si regions which are isolated from the p-type substrate by reversed-biased p-n diodes. An aluminum (Al) gate covers the region between source and drain, but is separated from the semiconductor by silicon dioxide (SiO_2). The complementary MOSFET is the p-type MOSFET. It contains p-type source and drain regions in an n-type substrate. The urge for faster and more complex circuits have driven the MOSFET since its intervention to ever smaller dimensions. For reasons of speed, compatibility with former process generations,... the decrease of the power supply voltage was not sufficiently in order to obtain constant internal electric fields. As a consequence, these internal fields increased and the energy of the carriers moving horizontally along the channel from source to drain increased continuously. High-energetic carriers are also called hot-carriers. Once the energy exceeds 1.1 eV, a collision with a silicon atom can generate an electron-hole pair. This process is called impact ionization. The created holes are collected by the substrate while most of the created electrons are attracted toward the drain junction. However, some electrons and holes can gain sufficient energy to surmount the Si-SiO_2 potential barrier and are injected into the gate oxide. These hot-carriers may generate damage to the substrate-insulator interface, but they also may be trapped in the oxide. This results in fixed charges in the oxide and electrically active states at the interface, inducing changes in key MOSFET parameters such as the threshold voltage. So, the oxide degradation leads to a deformation of the electrical characteristics of the MOSFET which has a negative influence on its functioning in the IC. When a certain level of oxide degradation is reached, the MOSFET is unable to execute this function and as a consequence, the total IC will fail [ACO96].

1.1.4.2. *Time Dependent Dielectric Breakdown (TDDB) of thin dielectrics:*

Thin dielectrics (mostly SiO_2) can be found between the two plates of a capacitor. TDDB of thin dielectrics is a major factor limiting the reliability of an IC. In a transistor, for example, the silicon dioxide layer can break down due to the high electrical fields applied between the metal layer and the semiconductor layer. It is now well established that breakdown occurs with time by the accumulation of charges in the dielectric layer. Many reliability stress measurements can be used for the reliability assessment of dielectrics [MAR98; DEG98a].

1.1.4.3. *Electrostatic Discharge (ESD):*

Sometimes, an electrical shock can be felt when touching a metal object after walking over a carpet or some other insulator. This event is an ESD event. ESD is defined as a rapid discharge event that transfers a finite amount of charge between two bodies at different potentials. For a person, ESD can cause not more than just a little pain, but it can destroy all but the most robust semiconductor devices [VIN98].

1.1.4.4. *Electromigration of on-chip interconnects:*

Electromigration is the most severe failure mechanism of on-chip interconnects. Such an interconnect is a conductor taking care of the electrical current transport inside a chip. Electromigration will be used in this thesis for bringing into practice most of the developed theories and will be discussed in detail.

The fact that electromigration can be responsible for an IC to fail is perfectly illustrated in a review paper of Hummel [HUM94]:

"..., the countdown of at least one space mission, originating from Cape Canaveral, was interrupted some years ago because of onboard computer failure. This occurrence was unmistakably traced back to a failure in thin metallic connection stripes of a microcircuit."

These on-chip interconnects are mostly made of Al doped with elements like Si or copper (Cu). These additional elements increase the lifetime of an interconnect because they slow down the diffusion of Al in the grain boundaries [AME70; NAH88]. With more recent technologies, layers are added on top of and/or below interconnects to stabilize and protect the surface. Such layers can be passivation and/or barrier layers. SiO_2 or silicon nitride (Si_4N_3) are often used for passivation. Refractory materials like Ti/TiN, TiW, WSi_2, etc. are often used as barrier layers. Passivation and barrier layers can substantially prolong the lifetime of an interconnect [NAH88; FAR87]. The replacement of Al by Cu as the basic metal for interconnects is one of the latest trends in interconnect technology [TOR96].

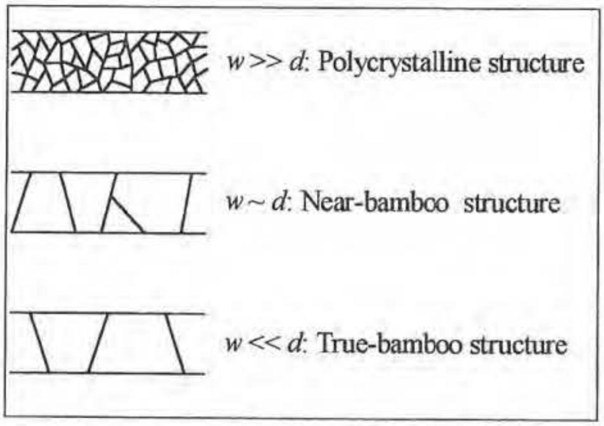

Figure 1.1: Influence of the ratio w/d on the grain structure of on-chip interconnects

As mentioned in section 1.1.2, the ongoing miniaturization in microelectronics requires that interconnects get more narrow. Different interconnect widths can result in a different line structure, which is mainly observed in the grain structure of these lines. In Figure 1.1, the influence of the ratio of the width of a line (w) and the average grain diameter (d) on the grain structure is given. The three most common types of grain structures are polycrystalline, near-bamboo and true-bamboo [THO93a]. They have the following characteristics:

- Polycrystalline:
  This structure is obtained when d is much smaller than w. A lot of grain boundaries and triple points (places where grain boundaries meet) are present in the interconnect.
- Near-bamboo:
  For these structures, d is in the same order of magnitude as w. Only few triple points can be found. Both bamboo and polycrystalline parts are observed.

- True-bamboo:
  Structures where d is much bigger than w. No triple points can be found along the line.

It is intuitively clear that interconnects with different grain structures can fail due to different causes. The ratio w/d has an influence on the lifetime of the interconnect. This will be discussed soon.

The basic principle of electromigration is common knowledge: when an electrical current is applied to a conductor, the ions of that conductor will diffuse in the direction of the electrons. This diffusion process is mainly the consequence of a momentum transfer between the electrons and the ions [MAL97]. This diffusion of metal ions can result in two types of damage in the conductor. First, holes, or voids, can be formed in the interconnect, leading to an open circuit. Second, material can accumulate into hillocks formed on top of the interconnect. This can lead to short a circuit. It has to be stressed that electromigration is a current driven process. This implies that interconnects which are subject to higher current densities are more sensitive to electromigration.

Ions in a conductor can diffuse via different paths. The three most common diffusion paths will be discussed now [SCH81].

- Grain boundary diffusion:
  The Al ions diffuse via the grain boundaries. For grain boundary diffusion to take place, triple points should be present in the interconnect. Hence, for true-bamboo structures, no grain boundary diffusion will take place. The activation energy (defined in app. A) for grain boundary diffusion is 0.4-0.5 eV.

- Bulk diffusion:
  The Al ions diffuse through the grains. The activation energy for bulk diffusion is higher than the activation energy for grain boundary diffusion: around 1.4 eV.

- Surface diffusion:
  The Al ions diffuse over the surface of the conductor. The activation energy for surface diffusion is around 0.3 eV. Surface diffusion can be avoided by putting a barrier layer on top of and below the interconnect. In practice, surface diffusion only takes place in the "catastrophic phase" of the aging process, when a void has already been formed. Then, the diffusion takes place via the void metal surface.

In real life, electromigration is a far more difficult process than theoretically described above. Especially when working with addition elements and layers placed on top of and/or below the line, the electromigration process becomes difficult to describe. A few items will be mentioned now in order to indicate the complexity of electromigration [HUM94]. First, multilayered structures; these are the result of a new technology where several layers of active and passive components are placed on top of each other. Tungsten (W) plugs are often used for the connection of two lines situated on different layers. In these plugs, electromigration does not occur. This leads to void formation near one end of the plug and to hillock formation near the other end. Second, Joule heating; this is defined as the heating of interconnects as a result of an electrical current. Joule heating leads to temperature gradients in a line. Texture is a third factor complicating the electromigration behavior of interconnects. Texture is the preferred orientation of certain lattice planes with respect to the plane of the IC and has an influence on electromigration resistance. Other factors worth mentioning here are that grains might grow due to Joule heating, that there is a chance that voids will move during electromigration, that the shape of voids depend on the grain structure of the line, etc.

1.1.4.5. Stress migration of on-chip interconnects:

Stress migration is the diffusion of Al ions as a result of mechanical stress and is another failure mechanism of on-chip interconnects. Stress gradients are a driving force for metal ion motion just like electromigration. Mechanical stress can be either tensile to form voids or compressive to form hillocks. Tensile stress makes diffusion easier while compressive stress makes diffusion more difficult [LLO97]. One possible cause for mechanical stress is the fact that the temperature coefficient of expansion for metal is much greater than that of the dielectrics and the Si to which it is attached. During IC processing, dielectrics are deposited at an elevated temperature. When the metal cools down, it wants to shrink much more than the Si or the oxide dielectric. However, since the metal adheres to the dielectrics and cannot shrink, it is left in a state of high tensile stress. The problem of stress migration becomes more severe in narrow lines, because mechanical stresses are higher in these lines.

1.2. Reliability experiment

In this section, the basic steps to be followed for performing reliability experiments are described. Examples are given for the most common components of an IC.

1.2.1. Basic steps for performing reliability experiments

For the determination of the reliability of some typical component of an IC (such as a new interconnect technology, a new type of MOSFET, an improved dielectric,...), so-called test structures are prepared. These test structures are especially designed to allow the measurement of some relevant characteristic(s) of the component under test. Such a characteristic is used for the determination of the component's lifetime. To study the aging process of on-chip interconnects, for example, the electrical resistance of the line is an important parameter. At the time the resistance drift exceeds a predefined value, the component is defined to have failed. Another example deals with dielectrics. For these structures, a characteristic of interest can be the leakage current. In the past, a dielectric has been defined to have failed at the time the leakage current shoots to infinity. This definition can not be applied for thinner dielectrics (t_{ox} < 5nm) anymore. A more extensive discussion of this topic will be given in section 1.2.2.2.

A major problem when testing the reliability of new components is that their lifetime under real life conditions is always extremely long (in the order of years). For that reason, the physical mechanisms that are responsible for a component to fail are studied and methods are established for accelerating these mechanisms. Next, the failure times of the devices operating under these accelerated conditions are measured and models are developed for extrapolating these results to real life conditions. Examples of the commonly used acceleration methods are given below.

In summary, when testing the reliability of some new technology, the next steps are to be followed.

- Develop a test structure which allows the measurement of some characteristic(s) of interest.
- Define the failure criterion (FC) based on these characteristic(s). A device under test is defined to have failed at the time it meets the FC.
- Choose some stress factors for accelerating the aging process(es) of the components.

- Measure the lifetimes of a number of components at several accelerated levels of each stress factor.
- Choose a model for extrapolating the failure times obtained at accelerated levels to real life conditions. Such a model consists of a life time distribution which models the scatter of the failure times and a relationship between these failure times and the applied stress.
- Accept or reject the new technology based on a predefined reliability criterion.

1.2.2. Examples of characteristics of interest, stress factors and lifetime models

Examples of the methodology described above are given for different elements of an IC. The electrical characteristics of interest, the stress factors and the acceleration models defined above are given for the investigation of the hot-carrier aging of MOSFET's, of the TDDB of dielectrics, of the response of an IC to an ESD event and of the electromigration failure behavior of on-chip interconnects.

1.2.2.1. Hot-carrier degradation of MOSFET's:

- Stress factor:
  When put under normal operation, the drain voltage, V_d, is set to 3.3 V. The aging processes of a MOSFET are accelerated when a higher V_d is applied.
- Characteristics of interest:
  In a typical reliability experiment, the gate voltage, V_g, is set to approximately $(V_d-1)/2$. In this situation, the substrate current, I_{SUB}, is maximal and the obtained lifetime of the MOSFET is worst-case. This means that when applying another V_g, the obtained lifetime is always higher than the one measured when applying $V_g = (V_d-1)/2$. For the determination of a MOSFET's lifetime, several parameters can be measured. A detailed description of these parameters is out of the scope of this thesis. The most frequently measured parameters are $I_{d,lin}$, V_t, $I_{d,sat}$, $G_{m,max}$ and I_{cp}. When put into operation, the MOSFET starts to degrade and the values for the above mentioned parameters start to shift. In case of an nMOSFET, the parameters $I_{d,lin}$, $I_{d,sat}$, I_{cp} and $G_{m,max}$ decrease while V_t increases. The definition of a failure is based on one of these 5 parameters. If $I_{d,lin}$, $I_{d,sat}$, I_{cp} or $G_{m,max}$ is chosen for this purpose, the MOSFET is usually defined to have failed when the drift of the chosen parameter exceeds a predefined drift value (e.g. 10 %). This is however not the case for V_t: the

MOSFET is defined to have failed when the absolute drift of V_t exceeds a predefined value (e.g. 10 mV).

- Models:

  Two models are often used in order to extrapolate the failure times obtained at high V_d's to real life conditions. First, the model of Takeda [TAK83] is an empirical model linking V_d and the median lifetime η as follows:

$$\eta = C * \exp(b / \mathrm{V_d}), \tag{1.1}$$

where C and b are two fitting parameters. Second, the model of Hu [ACO96], a semi-empirical model, links the median lifetime η with the substrate current I_{SUB} and the drain current I_d (both at time zero):

$$\frac{\eta * \mathrm{I_d}}{W} = C * \left(\frac{\mathrm{I_{SUB}}}{\mathrm{I_d}} \right)^{-b}, \tag{1.2}$$

where C and b are two fitting parameters. W is the channel width. Note that both I_{SUB} and I_d at time zero depend on V_d.

1.2.2.2. TDDB of dielectrics:

- Stress factors:

  Only the two most popular stress factors will be discussed [MAR98]. First, a constant voltage stress can be applied. Fields ranging from 3 to 12 MV/cm can be found in the literature. Measurement times range in the order of hours to months. A second type of stress which can be applied is a constant current stress. Current densities varying from 0.01 to 10,000 MA/cm^2 are applied. Besides a constant voltage or a constant current stress, temperature is another factor accelerating the breakdown process. In the literature, temperatures ranging from 25 to 400 °C are used. Mostly, however, measurements are performed at room temperature.

- Characteristics of interest:

  Depending on which type of stress is applied, a different type of characteristic is measured in order to determine the lifetime of the dielectric. When a constant voltage stress is applied, dielectric breakdown is detected when the measured current rises by several orders of magnitude and/or reaches a predefined breakdown current, I_{break}. Nowadays, for ultra thin dielectrics ($t_{ox} < 5nm$), a difference is made between soft and hard breakdown. For soft breakdown a device is defined to have failed when some level of noise is reached in the I(t)-characteristic. When a constant current stress is applied, the voltage typically drops at breakdown. For thin oxides ($t_{ox} < 5nm$), the breakdown event is no longer easy to identify and is often proceeded by noise which is a sign of stress-induced leakage current (SILC). For these oxides, either the destructive breakdown or a certain level of SILC can be the failure criterion [MAR98].

- Models:

Concerning the modeling of TDDB failure times, large disagreements can be found in the literature. Separate models have to be developed for intrinsic and extrinsic failures. An extrinsic failure is a failure due to some defect or impurity in the material, while an intrinsic failure is not defect-related. Both intrinsic and extrinsic failures can be modeled with the E-model and the 1/E-model. The E-model relates the median lifetime η with the applied electric field E (MV/cm) according to

$$\eta = C * \exp(\gamma * \mathrm{E}), \tag{1.3}$$

where C and γ are two fitting parameters.

For the 1/E-model, this relation is

$$\eta = C * \exp(G / \mathrm{E}) \tag{1.4}$$

with C and G two fitting parameters.

As mentioned above, large disagreements concerning this model can be found in the literature. In a paper of Martin et al. [MAR98], it is stated that no clearly defined and generally accepted model has been established. In papers of Degraeve et al. [DEG96; DEG98b] on the other hand, it is stated that the extrinsic failures should be fitted with the E-model, while the intrinsic failures can best be fitted with the 1/E-model, at least for dielectrics which are thicker than, say, 5 nm. In this paper, both a theoretical and an empirical motivation is given. For ultra-thin oxides, the disagreements that can be found in the literature are even higher than for thin oxides [WUN99].

Models for constant current stress experiments are similar to the models proposed above. Mostly, the voltage is assumed to be constant till breakdown occurs. In this case, the E-model and the 1/E-model can be used without modification.

A commonly used model for the temperature dependence of TDDB data is the Arrhenius model:

$$\eta = C * \exp\left(\frac{E_a}{\mathrm{k_B T}}\right) \tag{1.5}$$

C and E_a are two fitting parameters, k_B is the Boltzmann constant (= $8.6*10^{-5}$ eV/K) and T is the absolute temperature in degrees K. Although this model is often used, lot of people doubt its validity when applying it to failure times caused by TDDB [DIM99; PAN98].

1.2.2.3. Response of an IC to an the ESD event:

It is out of the scope of this thesis to extensively discuss the ESD process. For the more interested reader, we refer to the excellent review paper of Vinson et al. [VIN98]. Important to add is that engineers studying the ESD process mostly use one of these three models: the human body model, the machine model or the charged device model.

1.2.2.4. Electromigration failure behavior of interconnects:

- Stress factors:
  Two stress factors are used: temperature and current density increase. They both result in a higher mobility of the metal ions, making the line more susceptible to electromigration. Temperatures varying from 120 to 350 °C and current densities varying from 0.2 to 10 MA/cm^2 are applied. Operating conditions for temperature and current density range typically from 80 to 150 °C and from 0.2 to 1 MA/cm^2.

- Characteristic of interest:
  For monitoring the aging behavior of on-chip interconnects, the electrical resistance of the line is a widely used parameter. For older technologies, without barrier layers,

a line is defined to have failed when it breaks (infinite resistance). For newer technologies with passivation layer, it has been found that the conducting path is not immediately interrupted when the line breaks. The current path is temporarily diverted through the less conducting passivation layer. The definition of an 'open' as a FC is then no longer meaningful. Most investigators use therefore a resistance increase by 2-20 % from its original value for the definition of a failure [HUM94].

- Model:
  For the extrapolation from high to low levels, the Black-model [BLA69] is extensively used. This model relates the median lifetime η of a line with the temperature and the current density as follows:

$$\eta = C\left(\frac{1}{\mathrm{J}^n}\right)\exp\left(\frac{E_a}{\mathrm{k_B T}}\right), \qquad (1.6)$$

  where C is a materials constant, J is the current density in MA/cm^2, k_B the Boltzmann constant, T the absolute temperature in degrees K, E_a the activation energy of the thermally driven process in eV and n an exponent which usually has values between 1 and 3. More precise, the constant C depends on the geometry of the line, the physical characteristics of the film, the fact whether there is a passivation layer or not, the underlying substrate, etc. The fitting parameters of this model are C, E_a and n.

Note: While performing accelerated tests, it is advisable to choose the acceleration levels not too high. It might be possible that the failure mechanism of components stressed at high stress levels is different from the failure mechanism of components stressed at low levels. This would result in wrong predicted lifetimes at real life conditions.

1.2.3. Most common test strategies for on-chip interconnects

The currently used methods for testing the reliability of on-chip interconnects will be introduced. The MTTF test, the in-situ test and the 'fast' tests are described.

1.2.3.1. The MTTF test:

The measurement resolution of the measurement equipment used for performing MTTF (Median Time To Failure) tests is around 1 %. This implies that the FC, defined as the percentage resistance increase at which a line is found to have failed, cannot be chosen too low.

The stress levels of an MTTF test are between 170 °C and 240 °C for temperature stress and between 1 MA/cm^2 and 10 MA/cm^2 for current stress.

A major disadvantage of MTTF tests is that very high measuring times (around 3 months) are necessary in order to perform a statistically acceptable test. For the industry, this is not convenient, because for each small change in the production process of an interconnect, a new reliability experiment should be performed.

1.2.3.2. The in-situ test:

The in-situ test is a technique developed at the LUC in corporation with IMEC [DES97] and is nowadays commercialized by DESTIN N.V., a spin-off of the LUC and IMEC. The major strength of this method is that the resistance increase of a line is measured with a very high resolution (± 0.002 %). The high resolution is the consequence of two features. First, an extremely high temperature stability is obtained in the oven (a stability below 0.02 °C is reached). The second feature which results in the high resolution is the use of highly accurate measurement equipment.

The stress levels of an in-situ test are, for temperature, comparable to those applied in an MTTF test. For current density, however, these levels can be chosen lower than 1 MA/cm^2. These stress levels are still far above the operating conditions.

The major advantages of the in-situ test in comparison with the MTTF test are:

- The FC can be chosen very low. This decreases the total measurement time quite drastically.
- The resistance increase as a function of time can be followed with high precision. This offers the tool to analyze the physical processes responsible for the resistance to increase. Such an analysis is called a kinetic analysis.
- The problem of Joule heating, defined as the heating of interconnects as a result of electrical current flowing through the lines, becomes negligible when applying lower current stresses.

- Measurements can be performed closer to real life conditions. The chance of introducing other failure mechanisms than those occurring at real life conditions reduces.

The major disadvantage of the in-situ technique is that it might be dangerous to extrapolate the results to higher FC's.

<u>Note:</u> When measuring the reliability of semi-bamboo lines, it is advised to perform measurements at current densities close to real life conditions (from 0.2 to 1 MA/cm^2). For these lines, it is shown in a paper of Blech [BLE79] that both bulk and grain boundary diffusion can be responsible for a line to fail. The type of diffusion which is dominant depends on the maximum length l_{max} of the polycrystalline parts in the line and on the applied current density J. Blech has shown that when the product $J * l_{max}$ is smaller than some constant K, no electromigration will occur in the polycrystalline parts of the line (the constant K only depends on the type of interconnect). A consequence of this is that applying higher current densities than the operating current density is rather dangerous, because it might be possible that another type of diffusion is activated.

1.2.3.3. The 'fast' tests:

'Fast' tests [LLO92] are highly accelerated wafer-level tests such as SWEAT, BEM and ISOCURRENT. Here, current densities higher than 10 MA/cm^2 are applied. The lifetimes of the lines are very short (seconds to minutes). Applying external temperature stress is not necessary because only the Joule heating heats the lines to more than 175 °C.

Many authors have expressed doubt about the relevance of these 'fast' test results to predict the electromigration resistance of interconnects under real life conditions. In a review paper by Thompson and Lloyd [THO93b], it is stated in this way:

"A perception of relevance (for these tests) has been developed by correlations with life tests at lower current densities. Upon careful examination, these 'life tests' however, are often performed at current densities which promote temperature gradient failure (more than 2 MA/cm^2), and the correspondence is not surprising but nevertheless not of practical use."

1.3. Statistical analysis

In this section, an introduction of the most common techniques for statistically analyzing reliability experiments is given. A formal mathematical description of reliability experiments is proposed. The two most popular failure time distributions are described and the Maximum Likelihood Estimation technique is shortly introduced. An example of a recently performed reliability experiment is given.

1.3.1. Notations and assumptions concerning the setup of a reliability experiment

Before describing the assumptions made concerning the set up of the reliability experiments treated in this thesis, a few definitions have to be given. At the end of a reliability experiment, for each device under test (DUT), the event time t^e is registered. An event can be either a failure or a removal. As discussed in section 1.2, a DUT is defined to have failed at the time it meets a predefined FC. A DUT is considered to be a removal if it is removed from the experiment due to another reason than failure. Such reason can be: a technical defect of the measuring equipment, end of the test, etc. The event time corresponding to a failure is called a failure time, while the event time corresponding to a removal is defined to be a censoring time.

In this thesis, it is assumed that both the failure times of the failed DUT's and the censoring times of the removed DUT's are exactly known. So, the degradation behavior of each DUT is assumed to be continuously monitored. This is in contrast with interval monitoring, where each DUT is inspected for failure only after every predefined period of time (e.g. every 24 hours).

Further, it is assumed that the measurement of the degradation starts at the same time for all DUT's stressed at the same stress level. Also, the total measurement time of these DUT's is assumed to be equal. This total measurement time is specified at the beginning of the experiment. This type of experiment is called type I singly censored. In theory, we have for these experiments that all unfailed units have a common censoring time and that all failure times are lower than this censoring time. In practice, however, it might be possible that due to a defect of the measurement equipment, a number of removals occur before the last failure. This number is mostly small in comparison to the total number of failures. An example of a theoretical type I censored experiment is shown in Figure 1.2, while a more practical example is depicted in Figure 1.3.

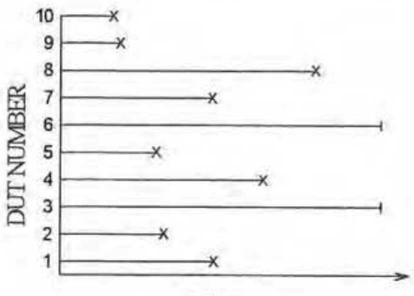

Figure 1.2: Example of a theoretical type I singly censored experiment. A X indicates a failure time and a | indicates a censoring time.

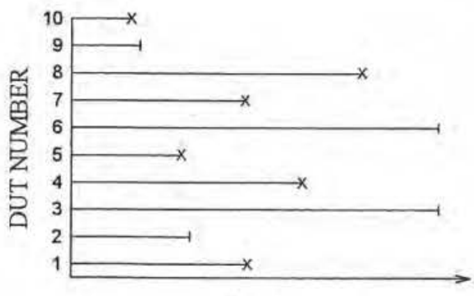

Figure 1.3: Example of a practical type I singly censored experiment. It can be observed that a number of removals occur before the last failure.

Besides type I singly censored experiments, type II singly censored and multiple censored experiments are the most popular. Type II singly censored experiments only differ from type I singly censored experiments in the specification of the total measurement time. A type II singly censored experiment is ended after the occurrence of a predefined number of failures. Preferably, type I experiments are performed because these experiments allow to control the total test time. This is, amongst others, important for economical reasons. Multiple censored data arise when DUT's go on test at different times. This, for example, happens when failed DUT's are replaced by new ones during the experiment. Type singly II censored and multiple censored data are not treated in this thesis, because they are not commonly used when performing reliability experiments.

A reliability experiment contains l stress factors. These stress factors are referred to as Ξ_1, Ξ_2, ..., Ξ_l. Measurements are performed at q stress levels. A stress level corresponds to one specific value for each Ξ_i, $i = 1, \ldots, l$. Thus, the j-th stress level is given by Ξ_{1j}, Ξ_{2j}, ..., Ξ_{lj}. The number of samples stressed at the j-th stress level is symbolized by N_j, while for the total stress time at this j-th level, the symbol T_j is used. At the end of this stress time, r_j samples have failed.

For extrapolating the results obtained at accelerated levels to real life conditions, the accelerating models for the median lifetime η described in section 1.2.2 are used. In general, such model describes the dependence of the median lifetime η on the l stress factors Ξ_i, $i = 1, \ldots, l$ and on the $l + 1$ fitting parameters Θ_0, Θ_1, ..., Θ_l leading to $\eta = \eta(\Xi_1, \Xi_2, \ldots, \Xi_l; \Theta_0, \Theta_1, \ldots, \Theta_l)$.

1.3.2. Example for an electromigration experiment

An electromigration experiment has 2 stress factors ($l = 2$): current density J and temperature T ($\Xi_1 = J$ and $\Xi_2 = T$). A typical type I singly censored experiment is summarized in Table 1.1. Note that especially the measuring times given in this table strongly depend on the technology under study. The model used for extrapolation to operating conditions is the Black-model given in formula (1.6). This model contains 3 fitting parameters. We have $\Theta_0 = C$, $\Theta_1 = n$ and $\Theta_2 = E_a$.

j ($q = 3$)	Ξ_{1j} (J in MA/cm^2)	Ξ_{2j} (T in °C)	N_j	T_j (in hours)
1	3	230	32	100
2	2	230	32	300
3	3	200	32	300

Table 1.1: Stress levels, number of samples and measuring times per stress level of a typical electromigration experiment with both current and temperature stress.

1.3.3. Failure time distributions

The distribution function of a random variable τ is defined as

$$\mathrm{F}(t) = P(\tau \le t)\,. \tag{1.7}$$

The density function is given by

$$\mathrm{f}(t) = \frac{d\mathrm{F}(t)}{dt}\,. \tag{1.8}$$

The hazard rate of a random variable τ is defined as

$$\lambda(t) = \frac{\mathrm{f}(t)}{1 - \mathrm{F}(t)}\,. \tag{1.9}$$

It is important to distinguish between hazard rate and density. For an item with a failure-free operating time t, the value $f(t)\delta t$ is for small δt the unconditional probability for failure in $(t , t+\delta t]$. On the other hand, the quantity $\lambda(t)\delta t$ is for small δt the conditional probability that the item will fail in the interval $(t , t+\delta t]$ given that it has not failed in $(0 , t]$.

The failure times per stress level are usually assumed to be lognormal or Weibull distributed. These distribution functions, which will be discussed below, have excellent performance in order to model the hazard rates. Attempts to fit the failure times to other failure time distributions than the Weibull or the lognormal can only rarely be found in the literature. For this reason, restriction to the lognormal and the Weibull distribution is made in this thesis.

1.3.3.1. The Weibull distribution

The positive random variable τ has a Weibull distribution if the distribution function has the following form:

$$\mathrm{F_W}(t,\eta,\sigma) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{1/\sigma}\right], \qquad t \geq 0, \quad \eta,\sigma > 0 . \tag{1.10}$$

The density function is given by

$$\mathrm{f_W}(t,\eta,\sigma) = \frac{1}{\eta\sigma}\left(\frac{t}{\eta}\right)^{\frac{1}{\sigma}-1} \exp\left[-\left(\frac{t}{\eta}\right)^{1/\sigma}\right], \tag{1.11}$$

and the hazard rate by

$$\lambda_{\mathrm{W}}(t,\eta,\sigma) = \frac{1}{\eta\sigma}\left(\frac{t}{\eta}\right)^{\frac{1}{\sigma}-1} . \tag{1.12}$$

η is often referred to as the characteristic life. It has the same unit as t. The dispersion parameter σ is unitless. $\sigma = 1$ yields the exponential distribution. For $\sigma < 1$, the hazard rate increases monotonically. For $\sigma > 1$, $\lambda_W(t)$ decreases monotonically. Notice that the Weibull distribution is often presented in terms of the shape parameter $\beta = 1/\sigma$. In this work, we have chosen for the σ-notation, because this is more consistent with the notation used for describing the lognormal distribution.

The Weibull distribution with $\beta > 1$ often occurs in applications as a distribution of the failure-free operating times of components which are subject to wear-out and/or fatigue [BIR94].

1.3.3.2. The lognormal distribution

The positive random variable τ has the lognormal distribution if its logarithm is normally distributed. For this lognormal distribution,

$$\mathrm{F_L}(t,\eta,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\int_0^t \frac{1}{x}\exp\left[-\left(\frac{\ln\left(x/\eta\right)^2}{2\sigma^2}\right)\right]dx \qquad t \geq 0, \quad \eta,\beta > 0\,. \tag{1.13}$$

The density function is given by

$$\mathrm{f_L}(t,\eta,\sigma) = \frac{1}{\sqrt{2\pi}\sigma t}\exp\left[-\left(\frac{\ln\left(t/\eta\right)^2}{2\sigma^2}\right)\right]. \tag{1.14}$$

The hazard rate can be calculated from (1.9). The median of this distribution is η and σ is the standard deviation of the log of t, or the log standard deviation. This σ will be referred to as the dispersion parameter, while the parameter $\beta = 1/\sigma$ will be referred to as the shape parameter. The density function of the lognormal distribution has the important property that it is practically zero at the origin, increases rapidly to a maximum and then decreases relatively quickly. The lognormal distribution is often used as a distribution function for the failure-free operating time of components in accelerated reliability testing as well as in cases where a large number of statistically independent random variables are combined together in a multiplicative fashion [BIR94].

1.3.4. Parameter estimation

1.3.4.1. Maximum Likelihood Estimation

For the lognormal distribution, the parameter η is equal to the time that 50 % of the components will have failed (the median). For the Weibull distribution, η equals the time that approximately 63 % will have failed. Although this is not completely true for Weibull distributions, the parameter η will in this thesis be referred to as the median life.

For the most common reliability tests, the dependence of η on the stress factors Ξ_1, Ξ_2, ..., Ξ_l is given in the acceleration models described in section 1.2. For estimating the lifetime of the components working under real life conditions, the fitting parameters of these acceleration models have to be estimated as well as the dispersion parameter σ of the distribution function of failure times.

In this thesis, the so-called Maximum Likelihood Estimator (MLE) has been used for parameter estimation. An extended discussion of the Maximum Likelihood method can be found in the book of Nelson [NEL90]. Before describing this technique, the event parameter δ^e has to be introduced. The 0-1 variable δ^e is defined to be 1 if the DUT has failed and 0 if it has been removed. So, a DUT is fully characterized when its event time t^e and its event parameter δ^e are known.

In principle, the Maximum Likelihood method maximizes the "total probability of the experiment". The "probability" of the k-th component stressed at stress level j to have an event at time $t^e_{j,k}$, with an event parameter $\delta^e_{j,k}$ is equal to

$$\pounds_{j,k} = \mathrm{f}(t^e_{j,k}, \eta_j, \sigma)^{\delta^e_{j,k}} \left[1 - \mathrm{F}(t^e_{j,k}, \eta_j, \sigma)\right]^{1-\delta^e_{j,k}}, \tag{1.15}$$

with $\eta_j = \eta(\Xi_{1,j}, ..., \Xi_{l,j}; \Theta_0, ..., \Theta_l)$. When the distribution function of the failure times is Weibull, F and f should be set to F_W and f_W, respectively. When, on the other hand, the underlying failure time distribution is lognormal, F and f are to be set to F_L and f_L, respectively.

The total probability, or the likelihood, of an experiment to have, for each component, exactly an event time $t^e_{j,k}$ and an event parameter $\delta^e_{j,k}$ equals

$$\pounds = \prod_{j=1}^{q} \prod_{k=1}^{N_j} \pounds_{j,k} \tag{1.16}$$

The values of the fitting parameters Θ_0, Θ_1, ..., Θ_l and the value of the dispersion parameter σ that maximizes this £, are defined to be the Maximum Likelihood Estimators (MLEs) of these parameters.

1.3.4.2. The software package FAILURE

The experiment types described in section 1.1.4 can be analyzed using the software package *FAILURE* [FAI97]. Using this package, the lifetime models described in section 1.2.2 can be fitted to the data using MLE. *FAILURE* was developed at the Interuniversity Micro-Electronics Center (IMEC) and is, within the framework of this thesis, translated to PC by the Institute of Materials Research (IMO) of the LUC. In the remaining part of this section, the abilities and the strengths of this software package will be enumerated. An example of how a statistical reliability analysis is typically performed is given as well.

FAILURE is an experiment oriented package. The different experiment types, the related stress factors and models included in *FAILURE* are given Table 1.2. This table contains an experiment type which has not been discussed in section 1.2: temperature storage. This experiment type has only one stress factor: temperature. When applying high temperatures, all kinds of diffusion processes can be accelerated.

Experiment type	**Stress factors**	**Models**
Electromigration	Temperature and current density	Black
TDDB	Constant voltage stress	E- and 1/E-model
Hot-carrier	Drain voltage	Takeda
Temperature storage	Temperature	Arrhenius

Table 1.2: Experiment types, stress factors and acceleration models incorporated in the FAILURE software

In *FAILURE*, the parameters of the models as given in Table 1.2, together with the shape parameter β (or the dispersion parameter $\sigma = 1/\beta$) of the underlying distribution of failure times can be estimated using MLE. Four distribution functions can be fitted: lognormal, Weibull, Normal and exponential. Three types of confidence bounds can be calculated, but an extended discussion of these confidence bounds is not the purpose of this thesis. This can be found in the manual of the software package.

The failure or censoring times for each separate DUT can easily be obtained in *FAILURE* on the base of measurement files with an easy file format. The FC can be chosen flexibly. Restricted data manipulation can be performed. For all fits used in this thesis, starting values for the fitting procedure are automatically set. The software package uses the Levenberg – Marquardt - Fletcher minimization [MAR63; FLE71] procedure in order to maximize the likelihood function (in practice, -Log(L) is minimized). This minimization procedure seldomly fails to converge.

Probability plots can easily be made. The model can be validated by comparing the fitted line with the raw data. Kolmogorov - Smirnov and chi-square goodness-of-fit tests can be used for model validation. Extrapolation to real life conditions can be done and the results can be printed and sent to the clipboard for input into other software packages.

1.3.4.3. A typical example of a reliability experiment

The results of measurements on commercial metal film resistors with 1% tolerance, a TCR below 10ppm/°C and 0.25 W power rating will be presented. The aging behavior of the DUT's was measured using the in-situ resistance measurement technique. Using this technique, we were able to continuously monitor, over time, the resistance change of the DUT's at high temperatures with a resolution in the order of 10^{-4} %. Note that the data presented here are a part of a larger data set presented at the CARTS-EUROPE'97 conference [CRO97] which is also published in Quality an Reliability Engineering International [CRO98a].

Samples at 3 different temperature levels were measured: 120, 145 and 155°C. At each level, 128 samples were measured. When all DUT's reached a drift of $5*10^{-2}$ %, the experiment was stopped. Some drift curves of the measurements performed at 155°C are shown in Figure 1.4.

Figure 1.5 shows the cumulative lognormal probability plot of the failure times using a FC of $5*10^{-2}$ %. Such lognormal probability plot shows the estimated percentage number of failures as a function of time. Each symbol type corresponds to one specific temperature condition. The axes of lognormal probability plots are scaled in such a way that if the points lay on a straight line, lognormality can be assumed. The same holds for Weibull probability plots. It can be observed that the data depicted in Figure 1.5 fit to a monomodal lognormal distribution function. The Arrhenius model was used in order to fit the temperature effect.

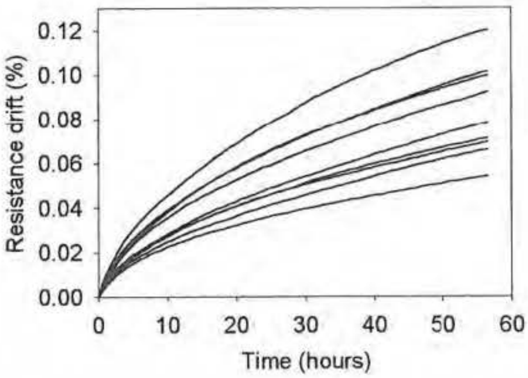

Figure 1.4: Degradation curves of some metal film resistors measured at 155°C.

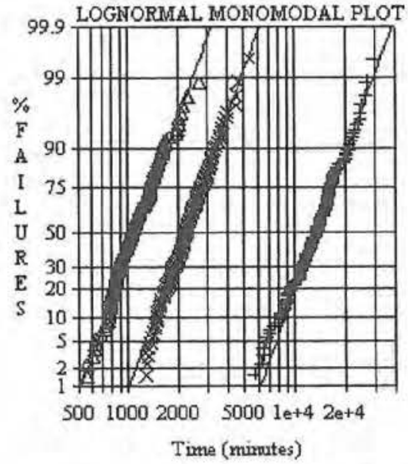

Figure 1.5: Lognormal probability plot of the failure times of the experiment presented above. (△ ≡ 155°C, x ≡ 145°C and + ≡ 120°C)

In Table 1.3, the MLE's of the parameters of the Arrhenius model and of the shape parameter β of the lognormal distribution function are given. Note that, instead of the parameter C, the median life η at temperature level 120 °C is given. This is done just because it is a more common way to represent fit results of reliability experiments.

Parameter	MLE
η @ 125°C	13500 min
E_a	1.03 eV
β	0.377

Table 1.3: MLE's of the model fitted to the data depicted in Figure 1.5

1.3.5. The choice of a suitable reliability measure

Until now, many industrial companies still work with the median life η as a reliability measure. This is not a good option. It is far better to work with low percentiles. A parameter of interest nowadays is the time at which x% of the components will have failed. The importance of choosing low percentiles instead of η as a reliability criterion can best be observed from the lines depicted on the lognormal probability plot of Figure 1.6.

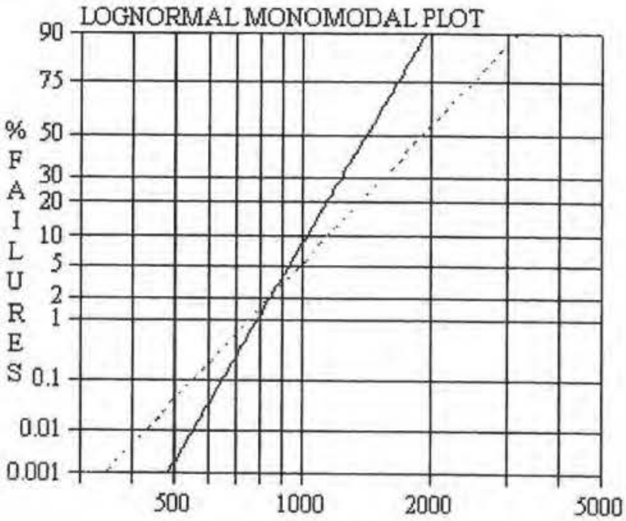

Figure 1.6: Lognormal probability plot of two lognormal distributions. The full line has median $\eta = 1400$ minutes and dispersion parameter $\sigma = 0.25$. For the dotted line, these values are $\eta = 1900$ minutes and $\sigma = 0.4$.

This figure shows the lognormal probability plot of two lognormal distributions. The solid line has a median life $\eta = 1400$ minutes and a dispersion parameter $\sigma = 0.25$. For the dotted line, these values are $\eta = 1900$ minutes and $\sigma = 0.4$. Purely based on η, the sample set corresponding to distribution of the dotted line is better. Due to a lower σ, the distribution corresponding to the solid line is better when using the, say, 0.01 % percentile as the reliability measure.

1.4. Definition of the problem

The problem this work deals with can best be defined as: HOW TO PLAN A TYPE I SINGLY CENSORED RELIABILITY EXPERIMENT SUCH THAT LOW PERCENTILES UNDER OPERATING CONDITIONS CAN BE ESTIMATED ACCURATELY? In this thesis, restriction will be made for reliability experiments with two stress factors. This is done for the following two reasons. First, the topic of planning experiments with only one stress factor has already been treated in several excellent papers. A short review of these papers will be given in chapter 2. Second, developing methods for planning experiments with more than two stress factors is not useful, simply because such experiments are rarely performed in practice. The development of measurement equipment and lifetime models for performing reliability experiments with three stress factors is time and money consuming, such that it is often found to be economically not acceptable to perform experiments with more than two stress factors.

In section 1.3, it was mentioned that at each j-th stress level of a type I singly censored reliability experiment N_j samples are stressed for a time T_j. It was also mentioned that in this thesis, only type I singly censored experiments will be considered, because type I censoring is most common in practice. Type II singly censoring, however, is more common in the theoretical literature, as it is mathematically more tractable [ESC86].

The aim of each reliability experiment is to estimate low percentiles under real life conditions. In section 1.3, type I singly censored reliability experiments are described using a number of parameters. These parameters can be divided into three groups:

- The stress levels for each stress factor: $\Xi_{i,j}$ ($i = 1$, ..., l ; $j = 1$,...., q). In this thesis, l will be restricted to 2.
- The number of components measured at each stress level: N_j ($j = 1$, ..., q).
- The measuring time at each stress level T_j ($j = 1$, ..., q).

During this work, the influence of these parameters on the estimates of low percentiles will be investigated. A mathematical technique will be developed for planning experiments which allow the accurate estimation of low percentiles under real life conditions. This technique is a merger, an extension and an improvement of two techniques proposed in the literature. Broadly speaking, the method comes down to minimizing the expected uncertainty of the low percentile of interest. Chapter 2 will be devoted to the complete description of the new technique and examples will be given as well.

Because the technique explained in chapter 2 is not robust against departures from the assumed stress-life relationship and the assumed model parameters, methods are proposed to account for them. A discussion of these methods and an analysis of the robustness of the proposed methods will be presented in chapter 3.

Concerning the technique discussed in chapters 2 and 3, two important remarks have to be made. These remarks are given in chapter 4 and are shortly enumerated here:

- It is assumed that the underlying distribution of failure times is known. In chapter 4, a technique is proposed for making the distinction between the lognormal and the Weibull distribution.
- It is assumed that the stress values actually applied are equal to the target stress values. In practical situations, this is never the case. The reason for this will be discussed and the influence of this on the estimates of low percentiles under real life conditions will be investigated.

2. A new technique for planning type I singly censored reliability experiments with two stress factors

In this chapter, a new method will be proposed for planning type I singly censored reliability experiments having two stress factors. Remember from section 1.4 that planning type I singly censored experiments comes down to (1) choosing the values for the applied stress, (2) setting the total measurement time per stress level and (3) deciding how to spread the total amount of DUT's over the different stress levels. A brief overview of the literature dealing with the topic of planning reliability experiments is given in section 2.1. The major shortcomings of the current techniques are enumerated and several ways of how to deal with them are discussed. All techniques discussed in section 2.1 as well as the new technique described in this chapter make use of the same concept for defining an optimum experimental plan. The purpose of an optimally planned experiment is to estimate important reliability parameters accurately. A generally used measure for the uncertainty of such an estimate is the so-called expected asymptotic variance (EAV). Section 2.2 deals with the idea of an EAV. The EAV of the estimate of all important reliability parameters will be calculated. In section 2.3, the new technique will be described. In general, this technique minimizes some linear combination of the EAV's of all important reliability parameters. Sections 2.2 and 2.3 have a mathematical background. Section 2.4 deals with several examples of how to use the new technique in practice. Conclusions are drawn in the last section of this chapter, section 2.5.

2.1. Literature

In this section, the literature dealing with the topic of planning reliability experiments are surveyed. The shortcomings of these papers are given and the way our technique deals with them is discussed.

2.1.1. Brief survey of literature

The references mentioned in this section assume that the data obtained from the planned experiment are fitted to the lifetime model using the method of Maximum Likelihood Estimation. Plans based on least squares estimation or best linear unbiased estimation will not be discussed, because they are less widespread and because the technique developed in this thesis is based on the Maximum Likelihood theory. Only the references dealing with type I singly censored experiments are discussed, because this is the only type of censoring that is considered in this thesis. Within this context, the papers that are worth to be mentioned are described now.

- In the paper of Kielpinski et al. [KIE75], optimum accelerated life test plans are presented for experiments with one stress factor where the failure times of the DUT's have a Normal or a lognormal distribution. A mathematical description of the theory can be found in a paper of Nelson et al. [NEL76]. The optimum accelerated life test plans are compared with so-called "traditional plans". These are plans with equal number of DUT's at equally spaced test stresses. The major conclusion of this paper is that it is best to perform tests at two stress levels with more DUT's at the lowest stress. The major drawback of these plans is that they are inefficient if the assumed lifetime model is incorrect [MEE84]. This is obviously because lifetime models can only be checked when the number of stress levels is higher than the number of fitting parameters contained in the model.
- The papers of Meeker et al. [MEE75] and Nelson et al. [NEL78] discuss optimum accelerated life test plans for experiments with one stress factor where the failure times of the DUT's have a Weibull distribution. The major conclusions are again that tests should be performed at two stress levels with more DUT's at the lowest stress. Again, the major drawback is that these plans are not robust against the incorrect assumption of the assumed lifetime model [MEE84].
- In a paper of Meeker et al. [MEE85], a test plan for experiments with one stress factor is proposed. Plans are given for both lognormal and Weibull distributed failure times. The plans considered here have three stress levels. Although these plans lack precision with respect to the uncertainty of the estimated reliability parameters, their ability to detect departures from the assumed lifetime model is higher.
- The paper of Yang [YAN94] deals with experiments with one stress factor. Lognormal and Weibull distributed failure times are both treated. The plans under investigation have four stress levels. In this paper, plans having different measurement times per stress level are allowed. Most papers referenced in this section [KIE75, NEL76, MEE75, NEL78, ESC95] choose equal measuring times at each stress level. This practice does not completely cover field

applications. A measurement time that is long enough to yield sufficient failures at the lowest stress is too long at the highest stress.

- Nelson [NEL90] describes a simulation-based method for evaluating and planning reliability experiments with more than one stress factor. Both lognormal and Weibull distributed failure times are considered. This method is useful for getting a first thought of how a proposed test plan will perform in reality. However, for the determination of an optimal plan, the method is far too laborious and time consuming for being useful in practice.
- Escobar et al. [ESC95] describes methods and guidelines for planning reliability experiments with two stress factors for models in which there is no interaction. Although this paper is the most useful in our situation, it has the great disadvantage that the plans assume an equal total measurement time at each stress level. As already mentioned above, this practice does not completely cover field applications. Another disadvantage of these methods is that a lot of statistical background is needed for using them, so that it is practically impossible to be used by reliability engineers without statistical background.

2.1.2. Major shortcomings of current techniques

The major shortcomings of the currently available techniques for planning type I singly censored reliability experiments are summarized in Table 2.1 and will be discussed now in turn. Observe that the properties of the new technique developed in this thesis are summarized in the last column of the table.

	[KIE75] [MEE75]	[MEE85]	[YAN94]	[NEL90]	[ESC95]	New method
Number of stress factors	1	1	1	2	2	2
Same stress time per stress level?	Yes	Yes	No	No	Yes	No
Minimize uncertainty of η or of percentile (p)?	η	η	η	p	p	p
Is method feasible in practice?	Yes	Yes	Yes	No	$\pm$	Yes
Control total test time?	Yes	Yes	No	Yes	Yes	Yes

Table 2.1: The major shortcomings of the techniques described in section 2.1.1. The last column indicates the properties of the new technique that will be described in sections 2.2 and 2.3.

- For the methods proposed in some papers, the number of stress factors is only allowed to be one ([KIE75; MEE75; MEE85; YAN94]). The techniques described in these papers perform well, but they cannot be used for planning reliability experiments with two stress factors.
- The practically-oriented requirement of allowing different measurement times per stress level is only fulfilled in two papers ([YAN94; NEL90]).
- Different criteria are used in order to define an optimum plan. In some papers ([KIE75; MEE75; MEE85; YAN94]), an optimum plan is defined as a plan which minimizes the uncertainty of the estimate of the median life η under real life conditions. In the other papers, the uncertainty of the estimate of some low percentile is used for the definition of an optimum plan. As already mentioned in section 1.3.5 of chapter 1, it is recommended to work with low percentiles.
- One of the methods is not practicable ([NEL90]), while another method can only be used by experts ([ESC95]).
- The total test time of the method proposed in one paper ([YAN94]) cannot be controlled. This is practically unacceptable for economical reasons.

The idea of the new technique proposed in this chapter is to get rid of all these shortcomings. Of course, the new technique has its own shortcomings. The purpose of chapter 3 is to account for them.

2.2. The concept of an asymptotic variance

All techniques discussed in section 2.1 as well as the new technique described here make use of the same concept for defining optimum plans. The purpose of an optimally planned experiment is to estimate important reliability parameters accurately. A generally used measure for the uncertainty of such an estimate is the so-called expected asymptotic variance (EAV) of this estimate. In this section, the concept of an EAV will be introduced. Next, the EAV of all important reliability parameters will be calculated making use of the so-called expected total Fisher information matrix $\mathcal{I}$ (simply pronounced as "I").

2.2.1. Concept of an asymptotic variance

As described in section 1.3.4, it is common practice to use the Maximum Likelihood Estimation technique for fitting the failure times coming from a reliability experiment to a certain distribution and a certain lifetime model. The parameters of this failure time distribution and this lifetime model are then estimated. These estimates are referred to as MLE's. The definition of a good test plan is one that keeps the expected uncertainty, or the expected variance, of these MLE's small.

Now, the expected variance of the MLE's of a type I singly censored experiment cannot be calculated analytically. That is why the expected *asymptotic* variance, EAV, of these estimates is often used for approximating these expected (true) variances. In theory, the maximum likelihood estimate of a parameter is, for a large number of failures, approximately Normal distributed with mean equal to the true value of that parameter and with variance equal to the EAV of the estimate. It is out of the scope of this text to give an extended description of the theory of the asymptotic behavior of MLE's. We will restrict ourselves to giving the analytical form of the EAV of the MLE's of the parameters of the failure time distribution and of the lifetime model. This analytical form is based on the so-called expected total Fisher information matrix $\mathcal{I}$. This $\mathcal{I}$ will be calculated below. Before this, some extra notation will be introduced.

2.2.2. Definitions and notation

For calculating the EAV of several important reliability parameters, it is convenient to describe the failure time distributions and the lifetime models discussed in chapter 1 in terms of the natural logarithm of time. The notation needed for the description of this transformation will be given in this section. First, the distribution of a random variable $Y = \ln(\tau)$ is described for τ being a Weibull or a lognormal distributed random variable. Then, the lifetime models discussed in section 1.2.2 will be transformed in terms of the $\ln(\eta)$.

When a random variable τ is Weibull distributed with median life η and dispersion parameter σ, it can be proven that the random variable $Y = \ln(\tau)$ follows the extreme value distribution with location parameter $\mu = \ln(\eta)$ and dispersion parameter σ. The distribution function of the extreme value distribution is

$$F_{EV}(y) = 1 - \exp\left[-\exp\left(\frac{y-\mu}{\sigma}\right)\right] \tag{2.1}$$

and its density function is given by

$$f_{EV}(y) = \frac{1}{\sigma}\exp\left[\frac{y-\mu}{\sigma} - \exp\left(\frac{y-\mu}{\sigma}\right)\right]. \tag{2.2}$$

It can also be shown that when a random variable τ is lognormal distributed with median life η and dispersion parameter σ, the random variable $Y = \ln(\tau)$ is Normal distributed with mean $\mu = \ln(\eta)$ and standard deviation σ (these μ and σ will be referred to as the location and the dispersion parameter, just as in the case of the extreme value distribution).

In section 1.2.2, lifetime models for the failure mechanisms of the most common components of an IC are given. Such a lifetime model describes the relation between the median life η of the underlying distribution function of failure times and the stress factors $\Xi_1, \Xi_2, ..., \Xi_l$. Such models depend on the fitting parameters $\Theta_0, \Theta_1, ..., \Theta_l$.

Define $\Xi_{i,H}$ as the highest possible value of the i-th stress factor. This value must be lower than the point that it causes other failure modes or that the lifetime model gets inadequate. This value can also be limited by the technical properties of the measurement equipment. Define $\Xi_{i,N}$ as the value of the i-th stress factor under real life conditions.

It can now easily be demonstrated that all models given in section 1.2.2 can be written in the following, linear, form:

$$\mu = \ln(\eta) = \theta_0 + \theta_1\xi_1 + \cdots + \theta_l\xi_l. \tag{2.3}$$

Here, the θ_i's are a function of the Θ_i's and the ξ_i's are a function of the Ξ_i's. Define $\xi_{i,H}$ and $\xi_{i,N}$ as the transformed values of $\Xi_{i,H}$ and $\Xi_{i,N}$, respectively. Without loss of generality, it can be demanded that $\xi_{i,H} = 0$ and $\xi_{i,N} = 1$ for $i = 1, ..., l$. How the transformation described in equation (2.3) should be performed in the particular case of Black's equation, is explained in the following section.

2.2.3. Linear form of Black's equation

As already described in section 1.2.2.4, Black's equation relates the median lifetime η of an on-chip interconnect to the temperature T and the current density J as follows:

$$\eta = C\left(\frac{1}{J^n}\right)\exp\left(\frac{E_a}{k_B T}\right), \tag{2.4}$$

For this equation, we have $\Xi_1 = J$, $\Xi_2 = T$, $\Theta_0 = C$, $\Theta_1 = n$ and $\Theta_2 = E_a$.

Replacing the stress levels J and T in equation (2.4) by their highest possible values J_H and T_H, gives:

$$\eta_H = C\left(\frac{1}{J_H^n}\right)\exp\left(\frac{E_a}{k_B T_H}\right) \tag{2.5}$$

Putting equations (2.4) and (2.5) together leads to:

$$\begin{aligned}
\mu = \ln(\eta) &= \ln(\eta_H) - n[\ln(J) - \ln(J_H)] + E_a\left(\frac{1}{k_B T} - \frac{1}{k_B T_H}\right) \\
&= \ln(\eta_H) - n[\ln(J_N) - \ln(J_H)]\left[\frac{\ln(J) - \ln(J_H)}{\ln(J_N) - \ln(J_H)}\right] + \frac{E_a}{k_B}\left(\frac{1}{T_N} - \frac{1}{T_H}\right)\left[\frac{\frac{1}{T} - \frac{1}{T_H}}{\frac{1}{T_N} - \frac{1}{T_H}}\right] \\
&= \theta_0 + \theta_1\xi_1 + \theta_2\xi_2
\end{aligned} \tag{2.6}$$

with

$$\theta_0 = \ln(\eta_H),$$

$$\theta_1 = -n[\ln(J_N) - \ln(j_H)],$$

$$\theta_2 = \frac{E_a}{k_B}\left(\frac{1}{T_N} - \frac{1}{T_H}\right),$$

$$\xi_1 = \frac{\ln(J) - \ln(J_H)}{\ln(J_N) - \ln(J_H)}$$

and

$$\xi_2 = \frac{\frac{1}{T} - \frac{1}{T_H}}{\frac{1}{T_N} - \frac{1}{T_H}}$$

so that $\xi_{i,H} = 0$ and $\xi_{i,N} = 1$ for $i = 1$ or 2. It is important to observe that the parameters θ_0, θ_1, θ_2, ξ_1 and ξ_2 are nothing but transformations of the physically relevant parameters η_H, n, E_a, J and T, respectively.

2.2.4. Fisher information matrix of a type I singly censored reliability experiment with two stress factors

In this section, the definition as well as the analytical form of the expected total Fisher information matrix $\mathcal{J}$ of type I singly censored experiments with two ($l = 2$) stress factors will be given. This will be done for experiments with either lognormal or Weibull distributed failure times. As mentioned above, this matrix $\mathcal{J}$ will be used for calculating the EAV of all important reliability parameters.

In the previous sections, the failure time distributions and the lifetime models have been rewritten in terms of the natural logarithm of time. Because $\mathcal{J}$ is based on the likelihood function $\mathcal{L}$ of an experiment, the description of the method of Maximum Likelihood Estimation, as presented in section 1.3.4, will shortly be repeated, but now in terms of log time.

Suppose that a type I singly censored experiment containing l stress factors ξ_i, $i = 1 ,..., l$ is performed. During this experiment, different samples are stressed at q stress levels $\xi_{i,j}$, $j = 1, ..., q$. At the j-th stress level, N_j samples are measured during a time T_j. Define $Y_j = \ln(T_j)$. For the k-th sample stressed at stress level j, the event time $t^e_{j,k}$, or $y^e_{j,k} = \ln\left(t^e_{j,k}\right)$, is registered together with the event parameter $\delta^e_{j,k}$ (1 if failed; 0 if censored). The probability for this event to occur at this event time is equal to

$$\mathcal{L}_{j,k} = \mathrm{f}(y^e_{j,k}, \mu_j, \sigma)^{\delta^e_{j,k}} \left[1 - \mathrm{F}(y^e_{j,k}, \mu_j, \sigma)\right]^{1-\delta^e_{j,k}}, \qquad (2.7)$$

with $\mu_j = \theta_0 + \theta_1\xi_{1,j} + ... + \theta_l\xi_{l,j}$. For Weibull distributed failure times, F and f should be set to F_{EV} and f_{EV}, respectively. When, on the other hand, the distribution function of failure times is lognormal, F and f should be set to the distribution or the density function of the Normal distribution.

The total probability $\pounds$ of this experiment to have for each k-th DUT stressed at the j-th stress level the event time $t^e_{j,k}$ and the event parameter $\delta^e_{j,k}$ equals

$$\pounds = \prod_{j=1}^{q}\prod_{k=1}^{N_j} \pounds_{j,k} \tag{2.8}$$

The maximum likelihood estimates of the parameters θ_0, θ_1, ..., θ_l and σ are defined as those values that maximize this $\pounds$.

Note that $\pounds$ is a function of $t^e_{j,k}$ and $\delta^e_{j,k}$ for $j = 1, .., q$ and $k = 1, ..., N_j$ and of the parameters θ_0, θ_1, ..., θ_l and σ.

Now, we will define the expected total Fisher information matrix $\mathcal{I}$. $\mathcal{I}$ is defined as minus the expectation of the matrix of the second derivatives of the likelihood function $\pounds$. For experiments with two stress factors, we have

$$\mathcal{I} = \begin{pmatrix} \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_0^2\right\} & \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_0\partial\theta_1\right\} & \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_0\partial\theta_2\right\} & \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_0\partial\sigma\right\} \\ & \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_1^2\right\} & \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_1\partial\theta_2\right\} & \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_1\partial\sigma\right\} \\ & & \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_2^2\right\} & \mathrm{E}\left\{-\partial^2 \pounds / \partial\theta_2\partial\sigma\right\} \\ & \text{symmetric} & & \mathrm{E}\left\{-\partial^2 \pounds / \partial\sigma^2\right\} \end{pmatrix} \tag{2.9}$$

The expectation of a function of a random vector $(X_1, X_2, ..., X_N)$, $G(X_1, X_2, ..., X_N)$, is defined as

$$E\{G(X_1,X_2,...,X_N)\} = \int...\int G(x_1,x_2,...,x_N) f_{X_1,X_2,...,X_N}(x_1,x_2,...,x_N)dx_1 dx_2...dx_N \tag{2.10}$$

with $f_{X_1,X_2,...,X_N}(X_1,X_2,...,X_N)$ the density function of $(X_1, X_2, ..., X_N)$.

It is very important to see that, due to this integration, the matrix $\mathcal{I}$ does not depend on the data $y^e_{j,k}$ and $\delta^e_{j,k}$. It only depends on the experimental setup: on the parameters θ_0, θ_1, θ_2 and σ, on the chosen stress levels $\xi_{i,j}$, on the chosen log total measurement times Y_j and on the number of DUT's, N_j. This fact implies that $\mathcal{I}$ can be calculated before the experiment has been performed. Notice that the toll that has to be paid here is the fact that the influence of the model assumptions increases. This is because $\mathcal{I}$ remains correct only when the correct model has been used in the integration.

Define, for the j-th stress level, z_j as

$$z_j = \frac{Y_j - \theta_0 - \theta_1\xi_{1,j} - \theta_2\xi_{2,j}}{\sigma}. \tag{2.11}$$

It can be calculated that the expected total Fisher information matrix $\mathcal{I}$ can be written in the following form:

$$\mathcal{I} = \frac{N}{\sigma^2}\begin{pmatrix} \sum_{j=1}^{q}\Pi_j A(z_j) & \sum_{j=1}^{q}\Pi_j\xi_{1,j}A(z_j) & \sum_{j=1}^{q}\Pi_j\xi_{2,j}A(z_j) & \sum_{j=1}^{q}\Pi_j B(z_j) \\ & \sum_{j=1}^{q}\Pi_j\xi_{1,j}^2 A(z_j) & \sum_{j=1}^{q}\Pi_j\xi_{1,j}\xi_{2,j}A(z_j) & \sum_{j=1}^{q}\Pi_j\xi_{1,j}B(z_j) \\ & & \sum_{j=1}^{q}\Pi_j\xi_{2,j}^2 A(z_j) & \sum_{j=1}^{q}\Pi_j\xi_{2,j}B(z_j) \\ & \text{symmetric} & & \sum_{j=1}^{q}\Pi_j C(z_j) \end{pmatrix} \tag{2.12}$$

with $N = \sum_{j=1}^{q} N_j$ and $\Pi_j = {N_j}/{N}$.

For Weibull distributed failure times, the functions A, B and C are defined as:

$$A(u) = 1 - \exp(-\exp(u))$$

$$B(u) = \int_0^{\exp(u)} v \ln(v) \exp(-v) \mathrm{d}v + u \exp(u) \exp(-\exp(u))$$

$$C(u) = 1 - \exp(-\exp(u)) + \int_0^{\exp(u)} v \ln(v)^2 \exp(-v) \mathrm{d}v + u^2 \exp(u) \exp(-\exp(u))$$

The obtained expected total Fisher information matrix is then defined as $\mathcal{I}_{\mathrm{W}}$.

For lognormal distributed failure times, the function A, B and C are defined as

$$A(u) = \Phi(u) + \lambda_{\mathrm{L}}(u)\varphi(u) - u\varphi(u)$$

$$B(u) = -\varphi(u)\left(1 - u\lambda_{\mathrm{L}}(u) + u^2\right)$$

$$C(u) = 2\Phi(u) - u\varphi(u)\left(1 + u^2 - u\lambda_{\mathrm{L}}(u)\right)$$

Here, $\Phi(u)$, $\varphi(u)$ and $\lambda_{\mathrm{L}}(u)$ are the standard Normal distribution, density and hazard function, respectively. The obtained expected total Fisher information matrix is defined as $\mathcal{I}_{\mathrm{L}}$.

2.2.5. EAV of several important reliability parameters

It has been described in the previous section how to calculate the total expected Fisher information matrix $\mathcal{I}$. Type I singly censored experiments with two stress factors having a Weibull or a lognormal underlying failure time distribution have been considered. It was noted that $\mathcal{I}$ only depends on the experimental setup and on the model parameters of the underlying failure time distribution and the lifetime model. In this section, $\mathcal{I}$ will be used for calculating the EAV of all important reliability parameters. Remember that this EAV will be used for approximating the expected (true) variance of the MLE of these parameters.

It can be proven that the inverse of the matrix $\mathcal{I}$ contains the expected asymptotic variances and covariances (EAV's and EACV's) of the four parameters in the model:

$$\mathcal{I}^{-1} = \begin{pmatrix} \mathrm{EAV}(\theta_0) & \mathrm{EACV}(\theta_0,\theta_1) & \mathrm{EACV}(\theta_0,\theta_2) & \mathrm{EACV}(\theta_0,\sigma) \\ & \mathrm{EAV}(\theta_1) & \mathrm{EACV}(\theta_1,\theta_2) & \mathrm{EACV}(\theta_1,\sigma) \\ & & \mathrm{EAV}(\theta_2) & \mathrm{EACV}(\theta_2,\sigma) \\ & \text{symmetric} & & \mathrm{EAV}(\sigma) \end{pmatrix} \tag{2.13}$$

This implies that the EAV of a linear combination $a_0\theta_0+a_1\theta_1+a_2\theta_2+a_3\sigma$ of the four model parameters of a type I singly censored reliability experiment with two stress factors is given by

$$\bar{a}\,\mathcal{I}^{-1}\,\bar{a}^{\mathrm{T}} \tag{2.14}$$

with $\bar{a} = (a_0 \quad a_1 \quad a_2 \quad a_3)$ and $\bar{a}^{\mathrm{T}}$ the transpose of $\bar{a}$. $\mathcal{I}$ should be set to $\mathcal{I}_W$ or $\mathcal{I}_L$ for experiments with Weibull or lognormal distributed failure times, respectively.

- Example 1:

  Since $\xi_{i,N} = 1$, $i = 1$ or 2, the location parameter μ_N under real life conditions is given by

$$\ln(\eta_N) = \mu_N = \theta_0 + \theta_1 + \theta_2 . \tag{2.15}$$

  This implies that for the calculation of the EAV of this parameter $\bar{a} = (1 \quad 1 \quad 1 \quad 0)$ has to be chosen.

  Making use of statistical property that both the Normal and the extreme value distribution are so-called location-dispersion distributions, it can be shown that the log p%-percentile at real life conditions, y_N^p, can be calculated by

$$y_N^p = \theta_0 + \theta_1 + \theta_2 + \sigma \mathrm{F}_{\mathrm{N}}^{-1}(p) . \tag{2.16}$$

For experiments with lognormal distributed failure times, $F_N^{-1}(p)$ is defined as the p%-percentile of the standard Normal distribution function. For experiments with Weibull distributed failure times, $F_N^{-1}(p)$ is defined as the p%-percentile of the standard extreme value distribution: $F_N^{-1}(p) = \ln(-\ln(1-p/100))$.

The EAV of y_N^p can be calculated using equation (2.14) with

$$\bar{a} = \begin{pmatrix} 1 & 1 & 1 & F_N^{-1}(p) \end{pmatrix}. \tag{2.17}$$

- Example 2:
  In the particular case of an electromigration experiment, the EAV of the activation energy E_a can be calculated using

$$\bar{a} = \begin{pmatrix} 0 & 0 & \left(\frac{1}{k_B}\left(\frac{1}{T_N} - \frac{1}{T_H}\right)\right)^{-1} & 0 \end{pmatrix}. \tag{2.18}$$

2.3. Planning experiments making use of the EAV

In the previous section, it is described how to calculate the EAV of the MLE's of all important reliability parameters GIVEN an experimental setup. It was important to see that these EAV's could be calculated BEFORE performing an experiment. This implies that these EAV's can be used for planning experiments.

In this section, the new technique for planning type I singly censored reliability experiments is described. First, it is shortly discussed how an optimal plan is defined in the papers of Yang [YAN94] and Escobar et al. [ESC95]. Then, all parameters necessary for defining an experimental plan are discussed. After these definitions, the so-called Optimum Plan Function (OPF), a function of all previously defined parameters, is created. The EAV's of important reliability parameters are used for defining this OPF. At the end of this section, the techniques used for finding the minimum of the OPF are discussed.

2.3.1. Literature

Most papers make use of the Maximum Likelihood technique for planning experiments. This is because this technique has several advantages in comparison with other techniques. The most important advantage is that ML estimates have a minimum variance for large sample sizes. For small sample sizes, the ML variance is generally comparable to those of other methods

In the paper of Yang [YAN94], in which plans having one stress factor and four stress levels were considered, an optimum plan is defined as a plan that minimizes

$$\left(C/\sigma^2\right)\mathrm{EAV}(\mu_N)+\left((1-C)/\eta_N\right)\sum_{j=1}^{q=4}T_j\,, \tag{2.19}$$

with C between 0 and 1.

In the paper of Escobar et al. [ESC95], two criteria are used for defining an optimum plan. As a primary criterion, the minimization of $\mathrm{EAV}(y_N^p)$ is proposed. As a secondary criterion, it is chosen to maximize the $\log_{10}$ of the determinant of the Fisher information matrix $\mathcal{I}$, $\log_{10}|\mathcal{I}|$, which is also known as D-optimality. This secondary criterion is motivated because the volume of an approximate joint confidence region for the model parameters is inversely proportional to an estimate of $\sqrt{|\mathcal{I}|}$.

The advantages and the disadvantages of both papers have been discussed in section 2.1.

2.3.2. Parameters defining an experimental plan

In this section, a summary of all parameters defining an experimental plan will be given. Most of these parameters have already been discussed. Some new parameters will be introduced in order to guarantee that the planned experiment can practically be carried out and statistically properly analyzed. The enumeration will contain fixed as well as unfixed parameters. The unfixed parameters will be transformed into 0-1 variables for reasons explained later on.

2.3.2.1. Enumeration of the parameters

A list of all parameters can be found in Table 2.2. As can be observed, these parameters are divided into four sets.

SET 1: Model specific parameters	
Parameters	*Remarks*
θ_0, θ_1, θ_2 and σ	These parameters have already been discussed.
Distribution function	Lognormal or Weibull.
SET 2: Experiment specific parameters	
Parameters	*Remarks*
TT	Total test time = $\sum_{j=1}^{q} T_j$.
N	Total number of DUT's = $\sum_{j=1}^{q} N_j$ in the experiment.
p	The p%-percentile under real life conditions is of interest.
SET 3: Parameters needed to guarantee that the statistical analysis is feasible	
Parameters	*Remarks*
MND	Minimum Number of DUT's measured per stress level. MND should be set between 0 and $1/q$.
MMNF	Minimum Mean Number of Failures. Expected number of DUT's to be failed per stress level should be at least MMNF.
MPT	Minimal Percentage Time: Minimal percentage of the total test time that should be devoted to each stress level. Note that MPT should be between 0% and $100/q$ %

(table is continued at the next page)

SET 4: Parameters related to the stress levels, the measurement time per stress level and the number of DUT's per stress level	
Parameters	*Remarks*
$\xi_{i,j}$	The values for the stress levels ($i = 1, 2$; $j = 1, ..., q$-1). We assume that $\xi_{i,q} = \xi_{i,H} = 0$ ($i = 1, 2$).
Y_j	Natural logarithm of the total measurement time at the j-th stress levels. Note that if Y_j is set for $j = 1, ..., q$-1, Y_q is set as well, since $\Sigma\exp(Y_j) = \mathrm{TT}$.
Π_j	$\Pi_j = N_j / N$. Note that if Π_j is set for $j = 1, ..., q$-1, Π_j is set as well, since $\Sigma\Pi_j = 1$.

Table 2.2: Enumeration of all parameters defining a type I singly censored test plan

The parameters contained in set 1 have already been discussed. The parameters θ_0, θ_1 and θ_2 define the relation between the location parameter μ and the stress factors ξ_1 and ξ_2. The parameter σ defines the dispersion parameter of the underlying distribution of failure times and is assumed to be independent of the applied stress.

The parameters of set 2 are experiment related. For economical reasons, it is demanded that the total test time and the total amount of DUT's to be stressed can be set in advance. The experimenters percentile of interest should also be set in advance.

When there are not enough failures at each stress level, the MLE's of the parameters θ_0, θ_1, θ_2 and σ may not exist. As a practical matter, a minimum number of failures should be maintained. In parameter set 3, the parameter MMNF (Minimum Mean Number of Failures) is introduced for guaranteeing that the planned experiment can statistically be properly analyzed. For the same reason, the parameters MND (Minimum Number of Devices) and MPT (Minimum Percentage Time) are introduced.

Set 4 contains the parameters that are related to the stress levels, to the measurement time per stress level and to the way how to spread the total amount of test structures over the different stress levels.

It is important to mention that the parameters defined in set 1 to 3 have to be set *before* planning the experiment. The parameters of set 1 depend on the type of DUT that is measured. The parameters defined in set 2 are to be controlled by the experimenters, while the parameters defined in set 3 have to be controlled from a statistical point of view. **So, only the parameters defined in set 4 can be left unfixed while defining an optimum experimental plan.**

2.3.2.2. Transformation of the unfixed parameters

In the next section, the optimum plan function (OPF), a function of all parameters given in the previous section will be defined. This OPF will be constructed so that the plan that corresponds to the minimum of this OPF is the optimum plan. In order to minimize the OPF using some numerical minimization procedure, it is convenient to transform the parameters defined in the previous section to values between 0 and 1. Since only parameter set 4 will be used in this minimization procedure, this transformation will only be applied to the parameters in that particular set. Remember that for each j-th stress level, parameter set 4 defines $\xi_{1,j}$, $\xi_{2,j}$, Y_j and Π_j.

We will now assume that one of the q stress levels equals $\xi_{i,H}$ for both i equal to 1 and 2. So, the optimum plan will allocate at least some DUT's to the factor-level combination in the experimental region that has the highest probability of failure. Nelson and Kielpinski [NEL76] give a heuristic argument for this assumption for experiments with one stress factor.

For notation-driven reasons, but without loss of generality, the q-th stress level will be defined as the highest possible level. So, $\xi_{i,q} = \xi_{i,H} = 0$ for $i = 1$ or 2.

Observe that if the parameters Y_j and Π_j are set for $j = 1, 2, ..., q$-1, the values for Y_q and Π_q are automatically known. This fact implies that the total number of parameters of set 4 equals 4*(q-1). Define $\overline{P}$ as a vector of length 4*(q-1), containing q-1 blocks of length 4. Each j-th block will contain the transformation to a value between 0 and 1 of the parameters $\xi_{1,j}$, $\xi_{2,j}$, Y_j and Π_j, in that order.

Let the first element of the j-th block contain the value $\xi_{1,j}$ and the second element of this block $\xi_{2,j}$. $\xi_{1,j}$ and $\xi_{2,j}$ are already values between 0 and 1.

The third element of the j-th block will contain the transformed value of Y_j. When introducing the parameter MPT (a parameter of set 3), it is automatically demanded that the measurement time at each stress level is at least TT*MPT/100. In order to guarantee this demand for each of the q stress levels, it is required that the measurement time per stress level, T_j, should be between TT*MPT/100 and TT-(q-1)*TT*MPT/100.

Transforming the log measurement time, Y_j, between 0 and 1 can for example be done as follows:

$$\ln(\mathrm{TT} * \mathrm{MPT}/100) \ < \ Y_j \ < \ \ln\left[\mathrm{TT} - (q-1) * \mathrm{TT} * \mathrm{MPT}/100\right]$$

or

$$0 \ < \ Y_j - \ln(\mathrm{TT} * \mathrm{MPT}/100) \ < \ \ln\left[\mathrm{TT} - (q-1) * \mathrm{TT} * \mathrm{MPT}/100\right] - \ln(\mathrm{TT} * \mathrm{MPT}/100)$$

or

$$0 \ < \ \frac{Y_j - \ln(\mathrm{TT} * \mathrm{MPT}/100)}{\ln\left[\mathrm{TT} - (q-1) * \mathrm{TT} * \mathrm{MPT}/100\right] - \ln(\mathrm{TT} * \mathrm{MPT}/100)} \ < \ 1$$

The fourth and last element of each j-th block will contain the transformed value of Π_j. The introduction of the parameter MND (a parameter of set 3) automatically demands that Π_j is higher than MND/N and lower than $(N\text{-}(q\text{-}1)\text{*MND})/N$. Transforming Π_j between 0 and 1 will be done as follows:

$$\mathrm{MND}/N \ < \ \Pi_j \ < \ \left(N - (q-1) * \mathrm{MND}\right)/N$$

or

$$0 \ < \ \Pi_j - \mathrm{MND}/N \ < \ \left(N - (q-1) * \mathrm{MND}\right)/N - \mathrm{MND}/N$$

or

$$0 \ < \ \frac{\Pi_j - \mathrm{MND}/N}{\left(N - (q-1) * \mathrm{MND}\right)/N - \mathrm{MND}/N} \ < \ 1$$

In summary, the vector $\overline{P}$ is defined as:

$$\overline{P} = \begin{pmatrix} \xi_{1,1} \\ \xi_{2,1} \\ \dfrac{Y_1 - \ln(\mathrm{TT} * \mathrm{MPT}/100)}{\ln[\mathrm{TT} - (q-1) * \mathrm{TT} * \mathrm{MPT}/100] - \ln(\mathrm{TT} * \mathrm{MPT}/100)} \\ \dfrac{\Pi_1 - \mathrm{MND}/N}{(N - (q-1) * \mathrm{MND})/N - \mathrm{MND}/N} \\ \vdots \\ \xi_{1,q-1} \\ \xi_{2,q-1} \\ \dfrac{Y_{q-1} - \ln(\mathrm{TT} * \mathrm{MPT}/100)}{\ln[\mathrm{TT} - (q-1) * \mathrm{TT} * \mathrm{MPT}/100] - \ln(\mathrm{TT} * \mathrm{MPT}/100)} \\ \dfrac{\Pi_{q-1} - \mathrm{MND}/N}{(N - (q-1) * \mathrm{MND})/N - \mathrm{MND}/N} \end{pmatrix} \tag{2.20}$$

This vector $\overline{P}$ contains the transformations of all parameters given in set 4. All elements of $\overline{P}$ are values between 0 and 1. However, these values can not vary freely between 0 and 1, but they are subjected to some restrictions. These restrictions will shortly be described in the next section.

2.3.2.3. Constraints on the components of $\overline{P}$

The values of $\overline{P}$ are subject to three different types of restrictions. These will be described now.

- The total measurement time at the q-th stress level, T_q, should be high enough. So, the log measurement times of the first q-1 stress levels have to fulfill the following requirement:

$$T_q = \mathrm{TT} - \sum_{j=1}^{q-1} \exp(Y_j) > \mathrm{MPT}/100 * \mathrm{TT} \tag{2.21}$$

- At least MND DUT's should be stressed at the q-th stress level. So, the fraction of DUT's stressed at the first q-1 stress levels should satisfy the following inequality:

$$\Pi_q = 1 - \sum_{j=1}^{q-1} \Pi_j > \frac{\text{MND}}{N} \tag{2.22}$$

- For each j-th stress level, the expected number of failures at the end of the test should be at least MMNF. For experiments with Weibull distributed failure times, the probability for a sample to have failed before log time Y_j is equal to $P_j = F_{EV}(Y_j\,;\,\mu_j, \sigma)$ with F_{EV} defined by equation (2.1). For experiments with a lognormal underlying failure time distribution, this probability equals $P_j = \Phi((Y_j - \mu)/\sigma)$, with Φ the standard Normal distribution function.
  So, the expected number of DUT's stressed at the j-th stress level that will have failed at the end of the test equals $N^*\Pi_j{}^*P_j$. This value should be higher than MMNF for each $j = 1, 2, ..., q$.

2.3.3. Definition of the Optimum Plan Function (OPF)

In the previous section, all parameters defining an experimental plan are enumerated. The unfixed parameters were re-scaled to values between 0 and 1 and a vector $\overline{P}$ of length $4^*(q\text{-}1)$ containing these re-scaled parameters was defined.

In this section, it will be discussed how to use the EAV's for the definition of optimum plans. The Optimum Plan Function (OPF), a function of all parameters given in the previous section will be defined. This OPF will be constructed so that the plan that corresponds to the minimum of this OPF is the optimum plan.

The OPF will be constructed as a weighted sum of six terms. In this section, each of these terms will be discussed and a motivation for each term will be given. Next, a way to weigh these terms will be proposed and at the end of this section, the OPF will be given in its complete form.

2.3.3.1. Discussion and motivation of six terms used for constructing the OPF

As stated above, the OPF will be constructed as a weighted sum of six terms. In this section, each of these six terms will be introduced and a motivation for each term will be given. Of course, many other terms can be proposed. Nevertheless, we feel that the six terms proposed here are the most relevant both from a practical and a theoretical point of view.

- Term 1-4:

  The first four terms are defined as the EAV's of the four parameters defining the lifetime model and the underlying failure time distribution: θ_0, θ_1, θ_2 and σ. So, define TERM(1)=EAV(θ_0), TERM(2)=EAV(θ_1), TERM(3)=EAV(θ_2) and TERM(4)=EAV(σ).

- Term 5:

  The fifth term will be defined as the EAV of the p%-percentile of the distribution function of failure times at real life conditions, y_N^p. So, $\text{TERM}(5) = \text{EAV}(y_N^p)$.

- Term 6:

  From equations (2.13), (2.14) and (2.17), it can be observed that TERM(5) consists of the weighted sum of the EAV's of the four parameters θ_0, θ_1, θ_2 and σ and a weighted sum of all possible expected asymptotic covariances between these four parameters. We have now experienced that the minimization of TERM(5) can become very difficult because the minimization procedure often fails to converge. We have found that this is due to the large number of terms contained in its sum. This is the major reason for introducing TERM(6). This term is defined as the part in the sum of TERM(5) containing only the terms with EAV's. So,

$$\text{TERM}(6) = \text{EAV}(\theta_0) + \text{EAV}(\theta_1) + \text{EAV}(\theta_2) + \text{F}_\text{N}^{-1}(p)^2 * \text{EAV}(\sigma) \qquad (2.23)$$

Besides the reason of making it easier to minimize, TERM(6) has been introduced since it is more appealing and intuitive for people having no statistical background. This is because this term neglects all expected asymptotic covariances. From a physical point of view, there is no "interaction" between the parameters $\theta_0, \theta_1, \theta_2$ and σ, so that for physicists, the existence of expected asymptotic covariances that are different from zero are counterintuitive. TERM(6) can best be interpreted as an "approximation" of the EAV of the p%-percentile of the distribution function of failure times at real life conditions, y_N^p.

2.3.3.2. Way to weigh each term

A rather naive way to define the OPF could be:

$$\mathrm{OPF}(\overline{P}) = \sum_{i=1}^{6} W(i)\mathrm{TERM}(i)\,, \tag{2.24}$$

with $\Sigma W(i) = 1$. The vector W can be set by the user. Note that the OPF is written as a function of $\overline{P}$. However, the OPF is also a function of parameter sets 1-3 defined in Table 2.2. The parameters in these three sets are not written in equation (2.24) since they are all fixed.

The different terms building up the OPF defined in equation (2.24) can vary several orders of magnitude. This is not a good start when it comes down to minimizing the OPF, because in that case, more attention will be paid to the terms with he highest values. It is better to divide each i-th term by a specific measure, MEAS(i), so that the ratio TERM(i)/MEAS(i) is in the same order of magnitude for each i. In this section, one such measure will be proposed.

The measure will be calculated using random $\overline{P}$ vectors, defined in formula (2.20). A random $\overline{P}$ vector can be obtained using the following strategy:

1. Simulate $4^*(q\text{-}1)$ uniform distributed values between 0 and 1.
2. Set each i-th element of $\overline{P}$ equal to the i-th simulated value.
3. If this $\overline{P}$ is in agreement with the constraints enumerated in section 2.3.2.3, this vector is a valid random $\overline{P}$ vector.
4. It the constraints are not fulfilled, go back to step 1.

The proposed measure for each term can be obtained by following the next steps:

1. Simulate 100 random vectors $\overline{P}$.
2. For each vector $\overline{P}$, calculate TERM(i) for $i = 1, \ldots, 6$ making use of equations (2.12) and (2.14).
3. Set MEAS(i) to the median of the TERM(i)'s for $i = 1, \ldots, 6$.

The choice of the value 100 as the number of random $\overline{P}$ vectors used for defining MEAS(i) for $i = 1, \ldots, 6$ is motivated in appendix C.

2.3.3.3. *The OPF*

Using the above described method for obtaining weights for the terms TERM(i) for $i = 1, \ldots, 6$, the OPF function is defined as

$$\mathrm{OPF}(\overline{P}) = \sum_{i=1}^{6} W(i)\,\frac{\mathrm{TERM}(i)}{\mathrm{MEAS}(i)} \tag{2.25}$$

By setting the vector W, the user will be able to plan an experiment that is in accordance with the specific purpose of the experiment. Note that most users will not set both $W(5)$ and $W(6)$ to a value different from 0, simply because TERM(6) is an approximation of TERM(5). Also note that this OPF depends on MEAS(i) and consequently on the 100 simulated $\overline{P}$ -vectors. Nevertheless, in section 2.4 and in appendix C, it will be shown that this dependence is negligible.

2.3.4. Minimizing the OPF

The calculation of the OPF described in the previous sections is performed using the statistical software package GAUSS [GAU97]. The integrals defined in section 2.2.4, needed for the calculation of the functions A, B and C for obtaining the matrix $\mathcal{I}_W$ are calculated using Simpson's rule [ABR70].

The minima of the OPF were obtained using the Constrained Optimization Application Module of GAUSS. This module offers the possibility to use the most powerful optimization techniques. It includes the Broyden, Fletcher, Goldfarb, Shanno method, the Davidon, Fletcher, Powell method and the constrained Newton-Raphson method. A short description of each method can be found in the manual of the module. A more extended description can be found in the work of Fletcher [FLE87]. During this work, it has been found that the Newton-Raphson method has the most rapid rate of convergence for the optimization problem considered here.

The choice of the starting values for the optimizer can be critical. It requires some practical experience to set the starting values so that the optimizer converges to the minimum of the OPF.

2.4. Examples

In this section, it is demonstrated how the new method can be applied in practice. Three examples are given. The differences between these examples are the choice of the parameters defined in sets 1-3 of Table 2.2 and the choice of the weight vector W. In the first two examples, experiments with 3 stress levels are planned. In example 3, experiments with 4 stress levels are considered.

2.4.1. Example 1

In this example, an experiment with 3 stress levels ($q = 3$) is planned. The underlying distribution function of the failure times is assumed to be Weibull. The parameters of parameter sets 1-3 of Table 2.2 are set to the values given in Table 2.3. For illustrative purposes, all parameter values are given in the specific case of Black's equation. These values have been chosen so that they are in accordance with practical situations.

Parameter	Value	Parameter	Value	Parameter	Value
T_H (°C)	300	n	2	TT (hours)	500
J_H (MA/cm²)	7	$\theta_1 = -n{*}\ln(J_N/J_H)$	7.11	N	128
T_N (°C)	125	E_a (eV)	0.65	p(%)	0.01
J_N (MA/cm²)	0.2	$\theta_2 = \frac{E_a}{k_B}\left(\frac{1}{T_N} - \frac{1}{T_H}\right)$	5.79	MND	16
η_H (hours)	40	σ	0.7	MMNF	8
$\theta_0 = \ln(\eta_H)$ (log-hours)	3.69	Distribution	Weibull	MPT (%)	1 (5 hours)

Table 2.3: Values of the parameters of parameter sets 1-3 used in this example.

The weight vector W is set to W = (0, 0.15, 0.15, 0, 0.7, 0). Using this weight function, most attention is paid to the fact that the 0.01%-percentile at real life conditions is accurately estimated. The activation energy E_a and the factor n of Black's equation also get special attention.

The vector MEAS is found to be MEAS = (x, 8.95, 5.59, x, 6.55, x). Notice that the values of the MEAS(i) for which W(i) = 0 are set to "x", because they are not relevant.

After minimizing the OPF, the optimum plan was obtained. This optimum plan is summarized in Table 2.4.

	Stress levels (j)		
	1	2	3
J_j (MA/cm^2)	1.98	7.00	7.00
$\xi_{1,j}$	0.36	0	0
T_j (°C)	300.0	209.8	300.0
$\xi_{2,j}$	0	0.42	0
T_j (hours)	222.3	203.1	74.6
N_j	47	41	40
η_j (hours)	502.4	468.1	40.0
ENF$_j$	12.6	10.8	36.2

Table 2.4: Optimum plan of the experiment proposed in this example.

EAV's		Appr. Conf. Int.	
Parameter	Value	Parameter	Value
θ_0 (log-hour^2)	0.014	η_H (hours)	40^{-8}_{+11}
θ_1	0.53	n	2.00 ± 0.41
θ_2	0.41	E_a (eV)	0.65 ± 0.14
σ	0.0063	σ	0.70 ± 0.16
$y_N^{0.01\%}$ (log-hour^2)	0.81	$t_N^{0.01\%}$ (years)	$2.9^{-2.4}_{+14.6}$

Table 2.5: Numerical evaluation of the plan proposed in Table 2.4. EAV's of θ_0, θ_1, θ_2, σ and $y_N^{0.01\%}$ and approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.01\%}$.

In the first row of this table, the current density J is given for each stress level. In the second row, the transformed value, ξ_1, of this current density is given. Remember from equation (2.6) that, for Black's equation, J and ξ_1 are related as follows:

$$\xi_1 = \frac{\ln(\mathrm{J}) - \ln(\mathrm{J}_H)}{\ln(\mathrm{J}_N) - \ln(\mathrm{J}_H)}.$$

The next two rows give the stress temperature T for each stress level, together with its transformed value ξ_2. From equation (2.6), we know that:

$$\xi_2 = \frac{\frac{1}{T} - \frac{1}{T_H}}{\frac{1}{T_N} - \frac{1}{T_H}}.$$

The fifth and the sixth row give the measurement time, T_j, and the number of DUT's per stress level, N_j, respectively.

The median life η, given in the seventh row, can be obtained using $\mu = \ln(\eta) = \theta_0 + \theta_1\xi_1 + \theta_2\xi_2$.

The last row of Table 2.4 gives the expected number of failures (ENF) per stress level. This ENF can be calculated using

$$\text{ENF}_j = \text{F}_\text{W}(T_j, \eta_j, \sigma = 0.7) * N_j,$$

where F_W is the Weibull distribution function given by equation (1.10).

The major conclusion with regard to Table 2.4 is the following: it is best to perform measurements at only two different temperature and current density levels. For both stress factors, it is advised to measure twice at the highest level and once at a lower level. When measuring at the lower level of the first stress factor, the stress level of the second stress factor should be set at its highest possible value, and vise versa. It is important to mention that the fact that it is best to perform measurements at only two different stress levels is in agreement with the one stress factor problem described in the papers of Kielpinski et al. [KIE75] and Meeker et al. [MEE75].

In these two papers, it is also stated that, for the one stress factor problem, more DUT's should be measured at low stresses than at high ones in order to obtain optimal plans. In these papers, however, it is assumed that the measurement times are equal for each stress level. As already mentioned in the beginning of this chapter, this practice does not cover field applications. From Table 2.4, is can be observed that, for the plan obtained in this example, the number of DUT's measured per stress level is about the same for each stress level and that the expected number of failures, ENF, is highest for the highest stress level.

It can also be observed from Table 2.4 that for the first stress level ($j = 1$), the measurement time, the number of measured DUT's and the expected number of failures are higher than for the second stress level ($j = 2$). Notice that at this first stress level, the DUT's are tested at the highest possible temperature and at a low current density, while for the second stress level, the opposite holds. So, it can in some way be stated that the first stress level is foreseen for the estimation of the current acceleration factor θ_1, while the second stress level is responsible for estimating the temperature acceleration factor θ_2. The difference in measurement time, the number of measured DUT's and the expected number of failures between these first two stress levels is consequently explained by the fact that θ_1 is higher than θ_2: the difference in log-lifetime between operating current density and the highest possible current density is higher than the difference in log-lifetime between the operating temperature and the highest possible temperature. So, in some sense, it is, "more difficult" to estimate θ_1 than to estimate θ_2.

In the first column of Table 2.5, the EAV's of the model parameters θ_0, θ_1, θ_2, σ and $y_N^{0.01\%}$ are given. In order to give an easier way to interpret these EAV's, approximate expected 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.01\%}$ have been calculated. These parameters are intuitively more accessible than the parameters θ_0, θ_1, θ_2, σ and $y_N^{0.01\%}$ and are presented in the second column of Table 2.5. Detailed information on how these 95% confidence intervals have been calculated can be found in appendix D.

From Table 2.5, it can be observed that, for the specific values of Table 2.3, the choice of TT = 500 hours and $N = 128$ is far too low. The resulting uncertainties on the estimates of the reliability parameters are unacceptably high. For the 0.01%-percentile under real life conditions, $t_N^{0.01\%}$, for example, the estimated 95% confidence interval is [2.9-2.4, 2.9+14.6] years = [0.5,17.5] years.

In the rest of this section, two extra subsections will be added. In the first one, several suggestions will be made for reducing the uncertainties shown in Table 2.5. In the second subsection, it will be examined how the obtained plan depends on the, by simulation obtained, vector MEAS.

- *Practical guidelines for improving the experimental plans:*

The purpose of this section is to give recommendations on how the uncertainties shown in Table 2.5 can be reduced. The effect of three possible suggestions will be investigated: 1) the increase of the total amount of measurement time, TT; 2) the increase of the total number of tested devices, N, and 3) the increase of the highest possible stress levels J_H

and T_H. The focus will in particular be on the uncertainty of the 0.01%-percentile under real life conditions, $t_N^{0.01\%}$.

Figure 2.1 shows the width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of the total measurement time TT for several values of the total number of DUT's N. For this figure, the highest possible conditions have been set to those given in Table 2.3: $J_H = 7\ MA/cm^2$ and $T_H = 300°C$. The experiment proposed in this example is indicated with the open dot. It can be observed that the increase of TT and N leads to a decrease of the width of the expected 95% confidence interval of $t_N^{0.01\%}$. This decrease is larger for lower values of N and TT. For $N = 80$, for example, a short calculation shows that when going from TT = 250 hours to 500 hours, the width decreases by a factor 2.7, while going from TT = 1000 hours to 2000 hours results in a decrease of only a factor 1.3. For N = 230, on the other hand, these two factors amounts to 1.9 and 1.2, respectively.

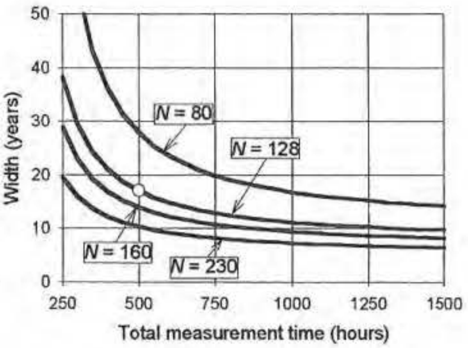

Figure 2.1: Width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of the total measurement time for several values of N. We have: $J_H = 7\ MA/cm^2$ and $T_H = 300°C$.

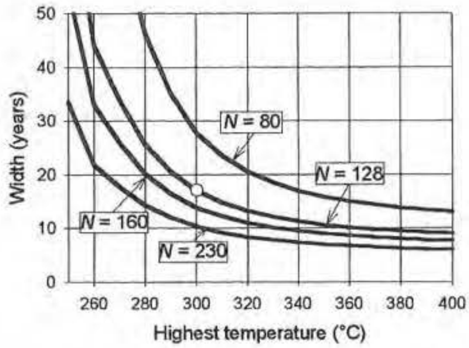

Figure 2.2: Width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of T_H for several values of N. We have: TT = 500 hours and $J_H = 7\ MA/cm^2$.

In figures 2.2 and 2.3, the effect of increasing the highest possible stress levels is investigated. Before discussing these plots, it is very important to notice that when increasing the highest possible stress levels, it always should be verified that the assumed lifetime model still holds for these high levels. For now, we will assume that this hypothesis holds.

Figure 2.2 shows the width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of the highest possible stress temperature T_H for several values of the total number of tested devices N. The total test time TT and the highest possible stress current density J_H have been set to those values given in Table 2.3: TT = 500 hours and $J_H = 7$ MA/cm^2.

Figure 2.3, on the other hand, investigates the width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of the highest possible stress temperature J_H. The total test time TT and the highest possible stress temperature T_H have again been set to the values given in Table 2.3: TT = 500 hours and T_H = 300°C.

From figures 2.2 and 2.3, it can be concluded that the uncertainty on $t_N^{0.01\%}$ is significantly decreased by increasing the highest possible stress levels. Again, this effect is higher for low values of N. For N = 80, for example, increasing T_H from 300°C to 400°C leads to a decrease of the width by a factor 2.1. For N = 230, this factor amounts to 1.7.

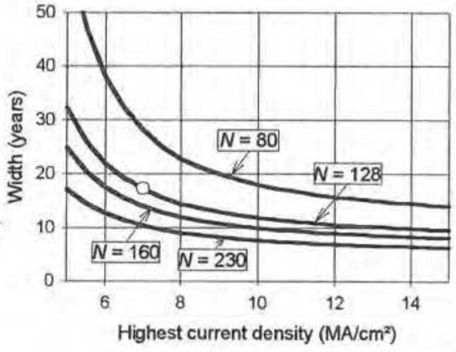

Figure 2.3: Width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of J_H for several values of N. We have: TT = 500 hours and T_H = 300°C.

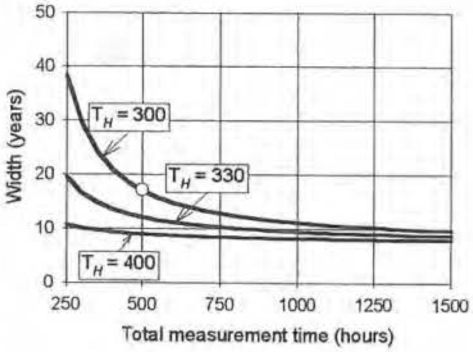

Figure 2.4: Width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of TT for several values of T_H. We have: N = 128 and J_H = 7 MA/cm^2.

Figure 2.4 gives the width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of the total measurement time TT for several values of the highest possible stress temperature T_H. Figure 2.5 again depicts this width as a function of the total measurement time but now for several values of the highest possible stress current density J_H. The last figure, Figure 2.6, shows this width as a function of T_H for several levels of J_H.

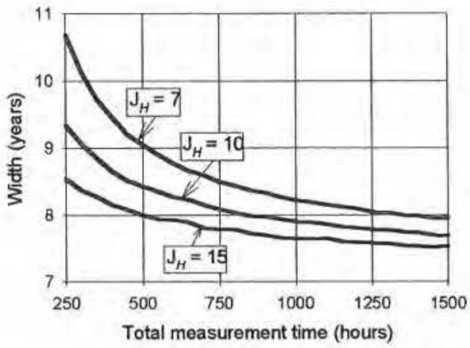

Figure 2.5: Width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of the total measurement time for several values of J_H. We have: N = 128 and T_H = 400°C.

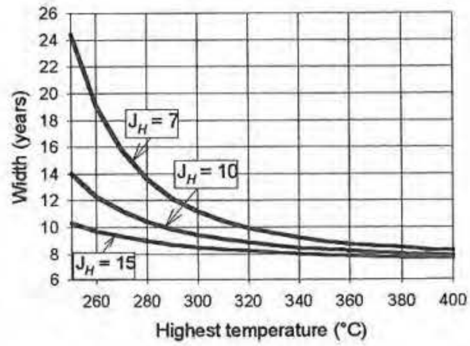

Figure 2.6: Width of the expected 95% confidence interval of $t_N^{0.01\%}$ as a function of T_H for several values of J_H. We have: N = 128 and TT=1000 hours.

Figures 2.1 – 2.6 have been made with the intention to give the reader a feeling for how the uncertainty on the predicted lifetime behaves as a function of the four investigated suggestions. Using these plots, the total test time, TT, the total number of tested devices, N, the highest possible stress temperature, T_H and the highest possible current density J_H can be set according to the specific requirements of the experiment. When, for example, a 95% confidence interval on $t_N^{0.01\%}$ with a width of maximally 10 years is wanted, all possible intersections of the depicted curves with the horizontal line "Width = 10 years" are possible solutions for this problem. Of course, it is up to the user to chose the experiment that is best in agreement with the available cost and total measurement time for the experiment.

In summary, the following practical guidelines for improving the experimental plans can be given. First, it has been observed that each of the four investigated suggestions proposed in this subsection lead to a significant decrease of the uncertainty of the predicted lifetime. Nevertheless, all four suggestions have their own disadvantages. The major disadvantage of increasing the total number of tested devices and the total measurement time is the fact that the total cost of the experiment increases. Increasing the highest possible temperature stress might require the purchase of new measurement equipment. The crucial disadvantage of increasing the highest possible current density stress originates from a more physical point of view: temperature gradients caused by very high current densities can lead to biased results.

The second practical guideline is the following: it has been observed that the effect of one suggestion is reduced if it is joined together with another one. The effect of increasing the total measurement time, for example, diminishes when the number of tested devices is increased as well. In terms of the plots depicted above: all curves come closer to each other at the right hand side of the figures. This can be interpreted as an interaction effect between the four factors considered here.

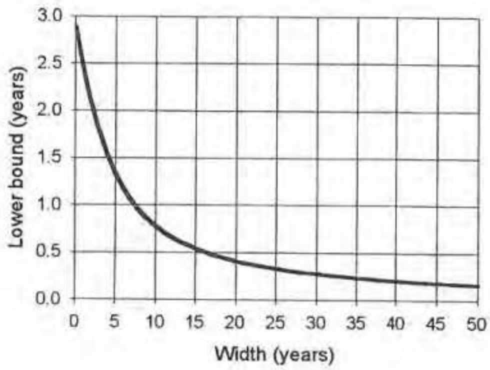

Figure 2.7: Relation between the width and the lower bound of the approximate 95% confidence interval on the predicted lifetime considered in this example.

At the end of this subsection, one important remark should be given. In figures 2.1 – 2.6, the total width of the expected 95% confidence interval on $t_N^{0.01\%}$ has been used as a measure of uncertainty. Some researchers, however, might be more interested in controlling only the lower bound of this 95% confidence interval. When, for example, the purpose of the planned experiment is to guarantee a certain lifetime, this lower bound might be more useful as a measure for the uncertainty on $t_N^{0.01\%}$. Since there is an unambiguous relation between the lower bound and the width of the expected confidence bound on $t_N^{0.01\%}$, figures 2.1 – 2.6 can also be used for this purpose. Figure 2.7 on the previous page shows the relation between the lower bound and the width of the expected 95% confidence interval on $t_N^{0.01\%}$. This figure has been created using the formulas given in appendix D.

- *Comment: The dependence of* MEAS *on the obtained plans:*

As described in section 2.3.3.2, the vector MEAS depends on random vectors $\overline{P}$. Remember that in this example, the vector MEAS has been set to MEAS = (x, 8.95, 5.59, x, 6.55, x). Of course, re-calculating MEAS using different vectors $\overline{P}$ lead to slightly different values of MEAS(i) for each i = 1,...,6. In appendix C, it is suggested that the number of simulated vectors $\overline{P}$ should be around 100 in order to have a vector MEAS which does not depend too much on the simulated vectors $\overline{P}$. This statement will be validated in this subsection by investigating the influence of different vectors MEAS on the obtained plans.

Table 2.6 on the next page summarizes the results of 5 planned experiments. The parameters of parameter sets 1-3 of Table 2.2 are the same for all these experiments and are set to the values given in Table 2.3. The differences between these plans are the values of the vector MEAS. For each plan, the vector MEAS is calculated using a different set of 100 simulated vectors $\overline{P}$. Note that plan 1 corresponds to the plan given in Table 2.4.

For each plan, the relevant elements of the vector MEAS are given. Remember that the vector W was set to (0,0.15,0.15,0,0.7,0). So, W(i) is different from 0 only when i is set to 2,3 or 5. Table 2.6 also shows the square root of the EAV of the parameters n, E_a and $y_N^{0.01\%}$ (for details on how to calculate these EAV's: see appendix D). The reason why only the parameters n, E_a and $y_N^{0.01\%}$ are considered here is obvious: by setting the vector W to (0, 0.15, 0.15, 0, 0.7, 0), most attention is paid to these three parameters. The last set of rows of Table 2.6 shows the stress levels, the measurement time and the number of measured DUT's for the two lowest stress levels.

It can be observed from this table that the dependence of the calculated vector MEAS on both the calculated standard deviations as well as on the parameters of the planned experiments is negligibly small. So, taking 100 simulated vectors $\overline{P}$ for calculating the vector MEAS is sufficient.

	Plan 1	Plan 2	Plan 3	Plan 4	Plan 5
MEAS(2)	8.95	7.69	9.96	8.98	11.78
MEAS(3)	5.59	4.30	6.62	5.14	6.61
MEAS(5)	6.55	5.32	8.76	6.85	6.40
$\sqrt{\mathrm{EAV}(n)}$	0.20	0.20	0.20	0.20	0.20
$\sqrt{\mathrm{EAV}(E_a)}$	0.072	0.071	0.071	0.071	0.072
$\sqrt{\mathrm{EAV}(y_N^{0.01\%})}$	0.90	0.90	0.90	0.90	0.90
$q = 1$					
Current density (MA/cm²)	1.98	1.98	1.99	1.98	1.96
Temperature(°C)	300.0	300.0	300.0	300.0	300.0
Test time (hours)	222.3	221.2	222.7	221.2	221.9
# of DUT's	47	47	47	47	47
$q = 2$					
Current density (MA/cm²)	7.00	7.00	7.00	7.00	7.00
Temperature(°C)	209.8	209.8	210.2	210.0	209.3
Test time (hours)	203.1	204.2	203.0	204.4	203.1
# of DUT's	41	41	41	41	42

Table 2.6: Optimum plans of the experiments proposed in this example. The differences between the plans are the calculated values of the vector MEAS.

2.4.2. Example 2

In this second example, another experiment with three stress levels is planned. The underlying distribution function of failure times is now assumed to be lognormal. The parameters of parameter sets 1-3 of Table 2.2 are set to the values given in Table 2.7.

Parameter	Value	Parameter	Value	Parameter	Value
T_H (°C)	250	n	1	TT (hours)	1000
J_H (MA/cm²)	5.5	$\theta_1 = -n*\ln(J_N/J_H)$	3.31	N	300
T_N (°C)	80	E_a (eV)	0.7	p (%)	0.1
J_N (MA/cm²)	0.2	$\theta_2 = \frac{E_a}{k_B}\left(\frac{1}{T_N} - \frac{1}{T_H}\right)$	7.48	MND	30
η_H (hours)	20	σ	0.5	MMNF	15
$\theta_0 = \ln(\eta_H)$ (log-hours)	3.00	Distribution	Lognormal	MPT (%)	0.5

Table 2.7: Values of the parameters of parameter sets 1-3 used in this example.

The weight vector W will be set to $W = (0, 0, 0, 0, 0, 1)$. Using this weight function, full attention is paid to the fact that the 0.1%-percentile under real life conditions will be accurately estimated. The "approximation" of the EAV of $y_N^{0.1\%}$ will be minimized. After minimizing the OPF, the optimum plan has been obtained. This optimum plan is summarized in Table 2.8.

From this table, it can again be concluded that it is best to perform measurements at only two different temperature and current density levels. This has already been reported in the previous example and is in agreement with the literature dealing with the one stress factor problem.

The conclusion in the papers of Kielpinski et al. [KIE75] and Meeker et al. [MEE75], that more DUT's should be measured at low stresses than at high ones in order to obtain optimal plans again does NOT hold for the two stress factor problem anymore. At the highest stress level ($j = 3$), it is advised to test 132 devices, while for the first stress level ($j = 1$), for example, only 59 DUT's should be measured. Also, the expected number of failures, ENF, is highest for the highest stress level. This fact can, as already mentioned in section 2.4.1, be explained by the fact that equal measurement times for each stress level were assumed in the papers of Kielpinski et al. and Meeker et al.

	Stress levels (j)		
	1	2	3
J_j (MA/cm^2)	0.33	5.50	5.50
$\xi_{1,j}$	0.85	0	0
T_j (°C)	250.0	155.9	250.0
$\xi_{2,j}$	0	0.46	0
T_j (hours)	329.6	621.3	49.2
N_j	59	109	132
η_j (hours)	333.4	603.5	20.0
ENF$_j$	29.0	57.0	127.3

Table 2.8: Optimum plan of the experiment proposed in this example.

EAV's		**Appr. Conf. Int.**	
Parameter	Value	Parameter	Value
θ_0 (log-hours)	0.0019	η_H (hours)	$20 \begin{matrix} -1.7 \\ +1.8 \end{matrix}$
θ_1	0.010	n	1.00 ± 0.061
θ_2	0.023	E_a (eV)	0.70 ± 0.028
σ	0.00064	σ	0.50 ± 0.051
$y_N^{0.1\%}$ (log-hours)	0.033	$t_N^{0.1\%}$ (years)	$24 \begin{matrix} -7.2 \\ +10.4 \end{matrix}$

Table 2.9: Numerical evaluation of the plan proposed in Table 2.8. EAV's θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ and approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.

In the previous example, it was mentioned that the measurement time, the number of measured DUT's and the expected number of failures of the first stress level were higher than those of the second stress level. This was explained by the fact that the parameter θ_1 was higher than the parameter θ_2. In this example, the opposite can be observed: θ_2 is higher than the parameter θ_1 and thus, the measurement time, the number of measured DUT's and the expected number of failures are higher for the stress level that is responsible for the estimation of θ_2.

Table 2.9 shows the EAV's of the model parameters θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$. The approximate expected asymptotic 95% confidence intervals of the parameters η_H, n, E_a, σ, $t_N^{0.1\%}$ are given as well. It can be observed that reasonable estimates of these parameters can be obtained by using the proposed plan. So, given the model parameters θ_0, θ_1, θ_2 and σ, the highest possible stress levels T_H and J_H and the operating conditions T_N and J_N, the choice of the total number of DUT's, $N = 300$, and the total measurement time, TT = 1000 hours, is sufficient.

Of course, the guidelines proposed at the end of the previous example can again be examined for the specific case of this example. This, however, will not be done because similar results are expected. However, one important comment is added: the influence of the choice of the weight vector W on the obtained plans is investigated.

- *Comment: The dependence of W(i):*

In this section, the dependence of the weight vector W on the obtained plans is investigated. As mentioned in section 2.3.3.2, by setting this W, the user is able to plan an experiment that is in accordance with the specific purpose of the experiment. Remember that in this example, the weight vector W has been set to $W = (0, 0, 0, 0, 0, 1)$.

Table 2.10 on the next page summarizes the results of 4 planned experiments. The parameters of sets 1-3 of Table 2.2 are the same for all these experiments and are set to the values given in Table 2.7. The difference between these plans is the choice of the weight vector W. Note that plan 1 corresponds to the plan given in Table 2.8.

It can be observed that, when paying more attention to estimating the parameter n, which is done by increasing $W(2)$, the lowest current density stress of the corresponding experimental plan decreases. The total measurement time and the number of DUT's stressed at that stress level will increase. The lowest temperature stress, on the other hand increases and the total measurement time and the number of stressed DUT's decrease.

The opposite happens holds when paying more attention to estimating the parameter E_a, which is done by increasing $W(3)$.

The effect of increasing $W(2)$ or $W(3)$ on $\sqrt{\mathrm{EAV}(n)}$ or $\sqrt{\mathrm{EAV}(E_a)}$, respectively, is not so clear. The following trend can be found: $\sqrt{\mathrm{EAV}(n)}$ decreases with increasing $W(2)$ and $\sqrt{\mathrm{EAV}(E_a)}$ decreases with increasing $W(3)$. Nevertheless, this relation is not so straightforward. This is probably due to the fact that the parameter $\sqrt{\mathrm{EAV}(y_N^{0.1\%})}$ is strongly correlated with $\sqrt{\mathrm{EAV}(n)}$ and $\sqrt{\mathrm{EAV}(E_a)}$.

	Plan 1	Plan 2	Plan 3	Plan 4
$W(2)$ (refers to n)	0	0.3	0	0.3
$W(3)$ (refers to Ea)	0	0	0.3	0.3
$W(6)$ (refers to $y_N^{0.1\%}$)	1	0.7	0.7	0.4
$\sqrt{\mathrm{EAV}(n)}$	0.030	0.023	0.033	0.024
$\sqrt{\mathrm{EAV}(E_a)}$	0.014	0.014	0.014	0.016
$\sqrt{\mathrm{EAV}(y_N^{0.1\%})}$	0.18	0.20	0.18	0.19
$q = 1$				
Current density (MA/cm²)	0.33	0.22	0.36	0.23
Temperature(°C)	250.0	250.0	250.0	250.0
Test time (hours)	330	491	288	462
# of DUT's	59	90	52	86
$q = 2$				
Current density (MA/cm²)	5.50	5.50	5.50	5.50
Temperature(°C)	155.9	161.3	154.0	159.3
Test time (hours)	621	466	666	499
# of DUT's	109	86	117	93

Table 2.10: Optimum plans of the experiments proposed in this example. The difference between the plans is in the choice of the weight vector W.

2.4.3. Example 3

An attempt has been made for minimizing the OPF for plans with more than 3 stress levels ($q > 3$). It turned out that these OPF's were difficult to minimize. It was found that, when reaching a minimum of the OPF with $q = 4$, the optimum plan converged to a plan with three stress levels ($q = 3$). This is in agreement with papers dealing with the one stress factor problem [KIE75; MEE75; MEE85; YAN94] and with the results found in the first two examples.

Notice that a crucial disadvantage of measuring at only 3 stress levels is the fact that the obtained plans are not able to detect departures from the assumed lifetime model and that they are not robust to departures from the assumed θ_0, θ_1, θ_2 and σ [MEE84]. This is obvious because a linear stress-life relationship can only be checked when measurements are performed at at least three different stress levels for each stress factor. This problem can be solved by *forcing* the experimental plans to have more than two stress levels for each stress factor. The purpose of chapter 3 is, amongst others, to take a closer look at this problem.

2.5. Conclusions

In this chapter, a method has been developed for planning type I singly censored experiments with two stress factors. The method is based on the expected asymptotic variance (EAV) of important reliability parameters. These EAV's could be calculated using the expected total Fisher information matrix $\mathcal{I}$. Making use of the EAV's, a so-called optimum plan function (OPF) was constructed.

The OPF was constructed as a weighted sum of 6 terms. All these terms were discussed in section 2.3.3.1. Each i-th term was weighted using a measure MEAS(i), so that the ratio TERM(i)/MEAS(i) was in the same order of magnitude for each i.

The vector W was used for weighting these 6 ratios. So, the OPF was defined as the sum of the 6 terms: $W(i)$*TERM(i)/MEAS(i) ($i = 1,...,6$). The weight vector W was introduced for allowing the user to plan experiments that are in accordance with the specific purpose of the experiment.

Two major conclusions concerning the obtained plans can be drawn. First, it is best to perform experiments at only 3 stress levels ($q = 3$) and to perform measurements at only two different levels for each stress factor. This is in agreement with the papers dealing with the one stress factor problem which are, amongst others, discussed in section 2.1. Another conclusion of these papers is that it is better to measure more DUT's at low stress levels than at high ones. In this chapter, it was found that this conclusion does not hold anymore for the two stress factor problem. This has been explained by the fact that our method does not assume equal measurement times per stress level.

Three important subsections were added to the example sections. First, several practical guidelines were given for improving the obtained plans. Questions like "What is the influence of changing the total measurement time on the estimated uncertainty of the reliability parameters?" were considered. Second, the influence of the vector MEAS on the obtained plans was investigated. It was found that when taking 100 simulated vectors $\overline{P}$ for calculating the vector MEAS, the influence of MEAS on the plans obtained was negligible. Third, the influence of the weight vector W was investigated. It was found that this vector behaved "as expected".

Three major shortcomings of this technique have to be mentioned. First, the obtained plans are not able to detect departures from the assumed lifetime model. This is obvious because a linear stress-life relationship can only be checked when measurements are performed at at least three different stress levels.

Second, the plans obtained are not robust to departures from the assumed θ_0, θ_1, θ_2 and σ. This is due to the fact that in the plans obtained, only 2 different stress levels per stress factor are foreseen [MEE84].

The third shortcoming originates from a more practical point of view. When having a closer look at the experimental plans given in Table 2.4 and Table 2.8, it can be observed that the measurement times of the two lowest stress levels can differ substantially. Sometimes, this might not be convenient from a practical point of view. It might be better to plan experiments for which these measurement times are equal. Such plans make it more easy to practically perform the planned experiment when more than one measurement oven is available.

3. A closer look at the shortcomings of the proposed technique

In this chapter, an attempt is made to solve the three most important shortcomings of the technique discussed in the previous chapter. Note that these shortcomings have already been mentioned at the end of the previous chapter.

As a running example throughout this chapter, the second example of chapter 2 will be used. This example has been treated in section 2.4.2. A closer look at the obtained plan will be taken in section 3.1.

The first shortcoming of the technique described in the previous chapter is that the obtained plans are not able to detect departures from the assumed lifetime model. This is obvious because a linear stress-life relationship can only be checked when measurements are performed at at least three different stress levels. In section 0, a method is proposed for *forcing* a plan to have three different stress levels for each stress factor.

The second shortcoming mentioned at the end of chapter 2, was that the obtained plans were not robust to departures from the assumed θ_0, θ_1, θ_2 and σ. According to Meeker [MEE84], this is due to the fact that the obtained plans only have 2 different stress levels per stress factor. In section 3.3, it is investigated whether the new plans, proposed in section 0, are less sensitive to departures from the assumed values of the model parameters.

The last shortcoming is that, for the planned experiments, the measurement times of the two lowest stress levels could differ substantially. Sometimes, this might not be convenient from a practical point of view. Section 3.4 proposes methods for planning experiments for which these two measurement times are equal.

In the final section of this chapter, some conclusions will be drawn.

3.1. A closer look at the second example of chapter 2

The second example of chapter 2, treated in section 2.4.2, will be used as a running example throughout this chapter. In this section, a closer look will be taken at the optimal plan obtained in that example. This plan will be further referred to as the "q3-plan", referring to its number of stress levels.

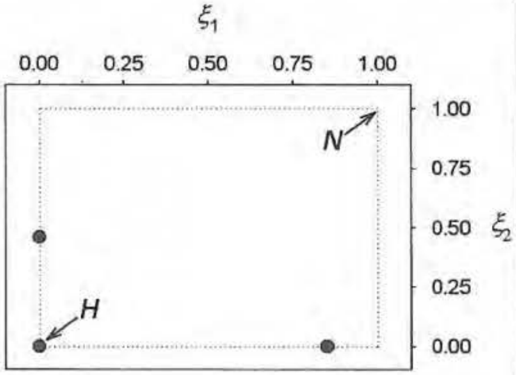

Figure 3.1: Schematic overview of the stress levels as planned in the q3-plan. Each circle corresponds to one stress level. The points labeled with "H" and "N", refer to the highest possible stress levels and the real life conditions, respectively.

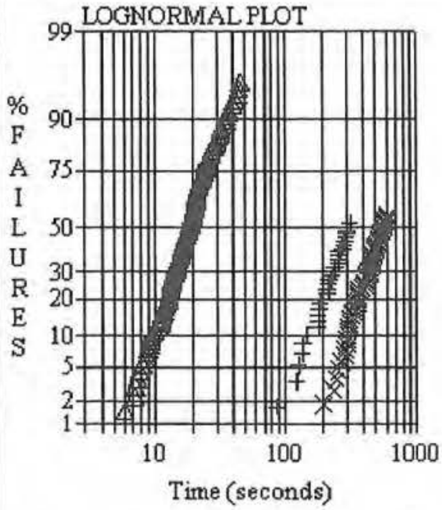

Figure 3.2: Lognormal probability plot of a simulated experiment with stress levels given in Figure 3.1. We have:
Δ : 250°C-5.5MA/cm²
+ : 250°C-0.33MA/cm²
X : 155.9°C-5.5MA/cm²

The values of the unfixed parameters on which the $q3$-plan was based can be found in table 2.7. A summary of the obtained optimal plan is given in table 2.8. The EAV's of the MLE's of the parameters θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ are given in table 2.9. In this table, approximate 95% confidence intervals of the intuitively more accessible parameters η_H, n, E_a, σ and $t_N^{0.1\%}$ are calculated as well.

A schematic overview of the stress levels of the $q3$-plan is depicted in Figure 3.1. Each circle corresponds to one particular stress level. The highest possible stress level is indicated with *"H"*, while the real life conditions are indicted with *"N"*. The planned experiment has been simulated using Monte Carlo simulation. A lognormal probability plot of such a simulated experiment is shown in Figure 3.2. Note that section 4.1 describes a method for simulating reliability experiments.

In order to give an indication of the importance of planning experiments, the $q3$-plan will now be compared with an experiment that has been planned without a formal optimization method. The stress levels, stress times and number of DUT's associated to each stress level of this experiment are given in Table 3.1. These values have been set without taking any optimization method into consideration.

Stress level	**Temperature** (°C)	**Current density** (MA/cm²)	**Test time** (hours)	**# of DUT's**	**ENF**
1	250	1.5	333	100	99.9
2	220	3.5	333	100	99.8
3	190	5.5	333	100	99.9

Table 3.1: The stress levels, stress times and number of DUT's associated to each stress level of an experiment that has been planned without taking any notification of any optimization method.

EAV's		**Appr. Conf. Int.**	
Parameter	Value	Parameter	Value
θ_0 (log-hours)	0.0019	η_H (hours)	$20^{-1.7}_{+1.8}$
θ_1	0.010	n	1.00 ± 0.061
θ_2	0.023	E_a (eV)	0.70 ± 0.028
σ	0.00064	σ	0.50 ± 0.051
$y_N^{0.1\%}$ (log-hours)	0.033	$t_N^{0.1\%}$ (years)	$24^{-7.2}_{+10.4}$

Table 3.2: Numerical evaluation of the q3-plan. EAV's of θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ and approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.

EAV's		**Appr. Conf. Int.**	
Parameter	Value	Parameter	Value
θ_0 (log-hours)	0.10	η_H (hours)	20^{-10}_{+18}
θ_1	0.73	n	1.0 ± 0.52
θ_2	1.61	E_a (eV)	0.7 ± 0.24
σ	0.0004	σ	0.5 ± 0.04
$y_N^{0.1\%}$ (log-hours)	3.23	$t_N^{0.1\%}$ (years)	24^{-23}_{+841}

Table 3.3: Numerical evaluation of the intuitively planned experiment. EAV's of θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ and approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.

The EAV's of the MLE's of the parameters θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ estimated from both the experiments following the $q3$-plan and the plan proposed in Table 3.1 can be found in Table 3.2 and Table 3.3, respectively. The approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$ are given as well. As can be observed, the expected 95% confidence intervals of the $q3$-plan are substantially narrower than those of the "intuitive" plan, although the ENF is higher for the latter. This fact underlines the importance of planning experiments: the statistical analysis of reliability experiments based on plans that have been planned without a formal method can result in an extremely uncertain estimate of the reliability of the tested product.

3.2. Forcing the planned experiments to stress at three different stress levels

One of the major drawbacks of the optimal plans proposed in chapter 2 is that they are not able to detect departures from the assumed lifetime model. This is obviously due to the fact that in the plans obtained, only 2 different stress levels per stress factor are foreseen.

In the third example of chapter 2, discussed in section 1.4.3, it has been mentioned that, when trying to find optimal plans with four stress levels ($q = 4$), the OPF automatically converged to plans having three stress levels ($q = 3$). In this section, methods will be proposed for forcing the optimal plan to have four or five different stress levels, such that the optimal plan will be forced to have at least three different levels for one of the two stress factors.This section will be split up into three parts. First, we will show how to force an optimal plan to have three different levels for the first stress factor. Then, the second stress factor will be forced to have three different stress levels. In the last part, a combination of the first two parts will be made: the optimal plan will be forced to have three different stress levels for each stress factor. In each part, the new plan will be compared with the $q3$-plan discussed in the section 3.1.

3.2.1. Three different stress levels for the first stress factor

The experiments planned in the first two examples of chapter 2 both have the same structure: at each i-th stress factor, the plan proposed to measure twice at $\xi_{i,H}$ and once at a lower level $\xi_{i,L}$. From table 2.8, it can be observed that for the $q3$-plan, we have

$\xi_{1,L} = 0.85$ and $\xi_{2,L} = 0.46$. A schematic overview of the 3 stress levels can be found in Figure 3.1.

The idea is now to enter a fourth stress level. For the first stress factor, the added stress level will be FORCED to be exactly between the lowest and the highest level. For the second stress factor, this added stress level will be forced to be equal to the highest level.

Corresponding to the restrictions mentioned above, the same OPF as the one leading to the $q3$-plan has been minimized. The optimal plan following from this minimization is given in Table 3.4. The EAV's of the MLE's of the parameters θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ are given in Table 3.5, together with the approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.

	Stress levels			
	1	2	3	4
J (MA/cm²)	0.35	5.50	5.50	1.40
ξ_1	0.83	0	0	0.41
T (°C)	250.0	156.8	250.0	250.0
ξ_2	0	0.45	0	0
Test time (h)	292.3	580.4	41.1	78.9
# DUT's	50	101	129	30
η (h)	310.9	579.5	20.0	78.9
ENF	22.7	50.73	119.3	15.0

Table 3.4: Optimum plan of the experiment proposed in this example.

EAV's		**Appr. Conf. Int.**	
Parameter	Value	Parameter	Value
θ_0 (log-hours)	0.0020	η_H (hours)	$20\,^{-1.7}_{+1.9}$
θ_1	0.012	n	1.00±0.066
θ_2	0.026	E_a	0.70±0.030
σ	0.00063	σ	0.50±0.050
$y_N^{0.1\%}$ (log-years)	0.038	$t_N^{0.1\%}$ (years)	$24\,^{-7.6}_{+11.3}$

Table 3.5: Numerical evaluation of the experiment proposed in Table 3.4. EAV's θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ and approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.

A schematic overview of the four stress levels is depicted in Figure 3.3. The full circles correspond to the optimally planned experiment depicted in Figure 3.1. The open circles correspond to the new plan. The new experiment has been simulated using Monte Carlo simulation. A lognormal probability plot of such a simulated experiment is shown in Figure 3.4.

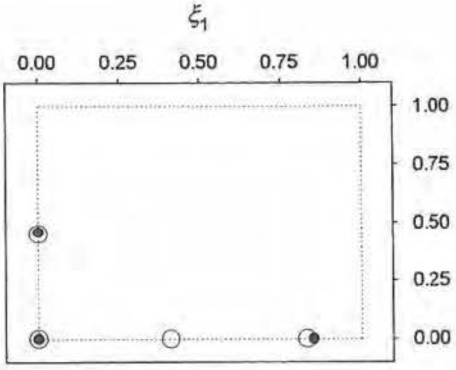

Figure 3.3: Schematic overview of the stress levels of the q3-plan (full circles) as well as the stress levels of the experiment planned in this section (open circles).

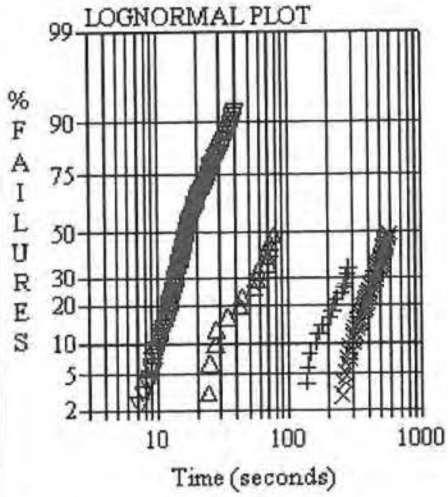

Figure 3.4: Lognormal probability plot of a simulated experiment as planned in Table 3.4. We have:
∇ : 250 °C-5.50 MA/cm²
Δ : 250 °C-1.40 MA/cm²
+ : 250.0 °C-0.35 MA/cm²
X : 156.8 °C-5.50 MA/cm²

It is now convenient to compare the new optimal plan with the unrestricted $q3$-plan. It can be observed that both plans do not differ substantially with respect to the choice of the stress levels, the choice of the measurement time per stress levels and the choice of the number of DUT's assigned to each stress level. The added stress level only has 30 DUT's assigned to it, which is exactly equal to the allowed minimum number of devices, MND. The total measurement time at that level has been set so that the expected number of failures is exactly equal to the minimum mean number of failures (remember from table 2.7 that MMNF has been set to 15). So, the new optimal plan is set in a way that "as few as possible" DUT's are assigned to the extra stress level and so that the proportion of the total measurement time TT assigned to this level is "as small as possible". This underlines the importance of setting the parameter MMNF high enough. Especially when the planned experiment is going to be used for model checking, the choice of, for example, MMNF = 3 might be too low, because such a low expected number of failures can never be enough for giving a clear indication of an incorrect lifetime model.

It can be observed that the 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$ are only a little wider for the new plan. So, when having doubt about the lifetime model concerning the first stress factor, which is the J-dependence in Black's equation for the example considered here, it is strongly advised to use the plan proposed in this section.

	Stress levels			
	1	2	3	4
J (MA/cm²)	0.35	5.50	5.50	5.50
ξ_1	0.83	0	0	0
T (°C)	250.0	158.1	250.0	199.7
ξ_2	0	0.44	0	0.22
Test time (h)	302.1	547.2	48.0	104.6
# DUT's	54	98	119	30
η (h)	316.7	547.2	20.0	104.6
ENF	24.9	48.5	113.8	15

Table 3.6: Optimum plan of the experiment proposed in this example.

EAV's		**Appr. Conf. Int.**	
Parameter	Value	Parameter	Value
θ_0 (log-hours)	0.0020	η_H (hours)	$20_{+1.9}^{-1.7}$
θ_1	0.012	n	1.00±0.065
θ_2	0.0.027	E_a (eV)	0.70±0.031
σ	0.00063	σ	0.50±0.050
$y_N^{0.1\%}$ (log-years)	0.038	$t_N^{0.1\%}$ (years)	$24_{+11.4}^{-7.7}$

Table 3.7: *Numerical evaluation of the plan proposed in Table 3.6. EAV's of θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ and approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.*

3.2.2. Three different stress levels for the second stress factor

In this section, a second optimal plan with four stress levels will be calculated. The second stress factor of the added stress level will be forced to be exactly between the lowest and the highest level. The stress level of the first stress factor will be set to its highest possible level.

Table 3.6 gives the optimal plan that minimizes the same OPF as the one leading to the $q3$-plan. The new minimization is in accordance with the restrictions mentioned above.

The EAV's of the MLE's of the parameters θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$, together with the approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$, can be found in Table 3.7.

A schematic overview of the stress levels of the new plan is shown in Figure 3.5 and a lognormal probability plot of a simulated experiment following the proposed plan is depicted in Figure 3.6.

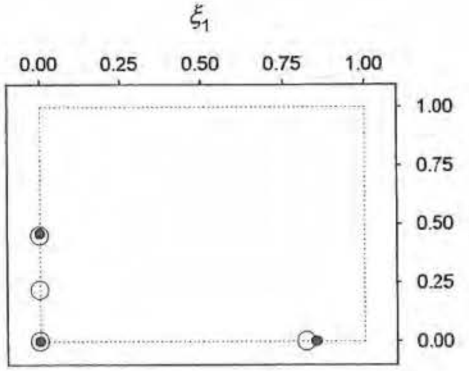

Figure 3.5: Schematic overview of the stress levels as planned in the q3-plan and the stress levels of the experiment planned in this section. The full circles correspond to the q3-experiment.

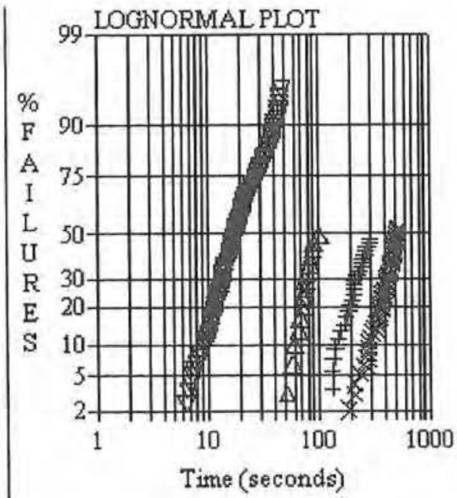

Figure 3.6: Lognormal probability plot of a simulated experiment as planned in Table 3.6. We have:
∇ : 250°C-5.50MA/cm²
Δ : 199.7°C-5.50MA/cm²
+ : 250.0°C-0.35MA/cm²
X : 158.1°C-5.50MA/cm²

From these tables and figures, it can again be concluded that the new plan does not differ substantially from the $q3$-plan proposed in chapter 2 with respect to the choice of the stress levels, the choice of the measurement time per stress levels and the choice of the number of DUT's assigned to each stress level. Again, the obtained estimated approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$ are not much wider for the new plan. So, when having doubt about the lifetime model concerning the second stress factor, which is the T-dependence in Black's equation for the example considered here, it is advised to use the plan proposed in this section.

3.2.3. Three different stress levels for both stress factors

In this section, both stress factors will be forced to have three different levels. This will be done by introducing a fifth stress level ($q = 5$). In comparison with the $q3$-plan, the first added stress level will, for the first stress factor, be forced to be between the lowest and the highest stress level, while the level of the second stress factor will be set to the highest possible stress level. The levels of the second added stress level will be forced to be as follows: highest possible level for the first stress factor and between the lowest and the highest level for the second stress factor.

Table 3.8 gives the optimal plan with five stress levels corresponding to the restrictions mentioned above. Table 3.9 contains the EAV's of the MLE's of the parameters θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$, as well as the approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.

	Stress levels				
	1	2	3	4	5
J (MA/cm²)	0.38	5.50	5.50	1.44	5.50
ξ_1	0.81	0	0	0.41	0
T (°C)	250.0	159.2	250.0	250.0	200.3
ξ_2	0	0.44	0	0	0.22
Test time (h)	266.6	507.5	47.1	76.6	102.2
# DUT's	45	90	105	30	30
η (h)	293.5	522.1	20.0	76.6	102.2
ENF	19.1	42.8	100.7	15.0	15.0

Table 3.8: Optimum plan of the experiment proposed in this example.

EAV's		Appr. Conf. Int.	
Parameter	Value	Parameter	Value
θ_0 (log-hours)	0.0021	η_H (hours)	$20\begin{matrix}-1.7\\+1.9\end{matrix}$
θ_1	0.014	n	1.00±0.071
θ_2	0.0.031	E_a (eV)	0.70±0.033
σ	0.00061	σ	0.50±0.049
$y_N^{0.1\%}$ (log-years)	0.045	$t_N^{0.1\%}$ (years)	$24\begin{matrix}-8.2\\+12.6\end{matrix}$

Table 3.9: Numerical evaluation of the plan proposed in Table 3.8. EAV's of θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ and approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.

In Figure 3.7, a schematic overview of the stress levels of the two stress factors is depicted. Figure 3.8 contains a lognormal probability plot of a simulated experiment following the proposed plan.

It can again be concluded that the new plan does not differ substantially from the $q3$-plan. When examining the obtained estimated approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$, it can again be concluded that when having doubt about Black's equation, it might be a good option to use the plan proposed in this section.

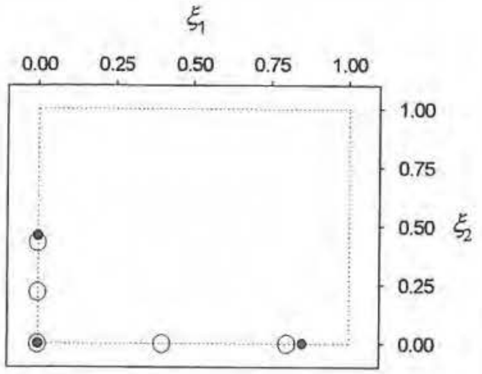

Figure 3.7: Schematic overview of the stress levels as planned in the q3-plan and the stress levels of the experiment planned in this section. The full circles correspond to the q3-experiment, while the open circles correspond to the new plan.

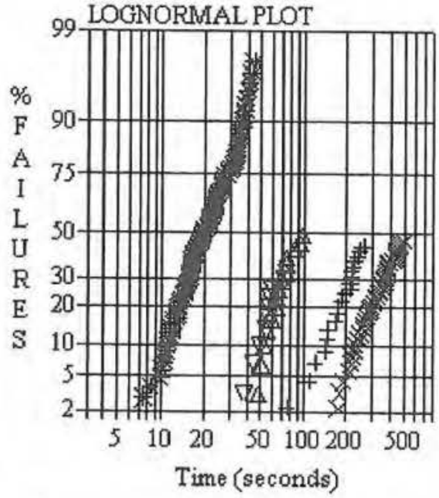

Figure 3.8: Lognormal probability plot of a simulated experiment with stress levels depicted in Figure 3.7. The symbols refer to the following stress levels:

✱ : 250°C-5.50 MA/cm²
∇ : 200.3°C-5.50MA/cm²
Δ : 250.0°C-1.44MA/cm²
+ : 250.0°C-0.38MA/cm²
X : 159.2°C-5.50 MA/cm²

3.3. Sensitivity of the new plans to misspecified values of the model parameters

In practice, θ_0, θ_1, θ_2 and σ are unknown at the test planning stage. However, it is generally possible to use some combination of engineering judgment, design specifications and information on similar products to obtain a range of possible values for these parameters. One can then evaluate test plans over that range to find a plan that is generally satisfactory.

In this section, the new plans proposed in this chapter are compared with the $q3$-plan with respect to their sensitivity to misspecified values of the model parameters θ_0, θ_1, θ_2 and σ. As a measure for such sensitivity, two different criteria are used. Before defining these criteria, some theory discussed in chapter 2 is briefly restated and some extra notation is introduced.

In section 2.3.3.2., the optimum plan function OPF has been written as a function of the vector $\overline{P}$. However, this OPF is also a function of the model parameters θ_0, θ_1, θ_2 and σ. In chapter 2, these model parameters have been considered to be fixed. Since in this section the influence of misspecified model parameters will be investigated, the OPF will also be written as a function of the model parameters θ_0, θ_1, θ_2 and σ. Define the vector $\overline{\theta}$ as a vector of length 4, each element containing one model parameter:

$$\overline{\theta} = (\theta_0 \quad \theta_1 \quad \theta_2 \quad \sigma) \tag{3.1}$$

So, the OPF will be written as $\mathrm{OPF}_{\overline{\theta}}(\overline{P})$. Now, define the vector $\overline{P}_{\overline{\theta}}$ as the vector that minimizes this OPF. Observe that $\overline{P}_{\overline{\theta}}$ is in fact nothing else but the optimum plan. The question this section deals with is: How does this plan $\overline{P}_{\overline{\theta}}$ change when the vector of model parameter changes from $\overline{\theta}$ to $\overline{\theta}'$? Define the vector $\overline{P}_{\overline{\theta}'}$ as the vector that minimizes $\mathrm{OPF}_{\overline{\theta}'}(\overline{P})$.

Now, the two criteria for quantifying the sensitivity of the proposed plans will be introduced. The first criterion has been proposed in the paper of Meeker [MEE84] and is also used in the paper of Yang [YAN94]. The following criterion has been used:

$$\mathrm{CRITERION1} = \frac{\mathrm{OPF}_{\overline{\theta}}(\overline{P})}{\mathrm{OPF}_{\overline{\theta}'}(\overline{P})} \tag{3.2}$$

For this criterion, the following rule holds: the closer the obtained ratio is to 1, the less sensitive the obtained plan is to the choice of the vector $\overline{\theta}$.

The second criterion is more or less based on a paper of Cook [COO86]. We propose the following criterion:

$$\mathrm{CRITERION2} = {\left\|\overline{P}_{\overline{\theta}} - \overline{P}_{\overline{\theta}'}\right\|^2}\Big/{m} \tag{3.3}$$

where m is the length of the vector $\overline{P}$. The operation $\| \; \|$ is the length of this vector in the m-dimensional space. For this criterion, the following rule applies: the closer this length is to zero, the less sensitive the obtained plan is to the choice of the vector $\overline{\theta}$.

Table 3.10 evaluates the sensitivity to misspecifications of the model parameters θ_0, θ_1, θ_2 and σ of two plans: the $q3$-plan and the plan with 5 stress levels, proposed in section 3.2.3. This last plan will further be referred to as the $q5$-plan. Each row of Table 3.10 shows the calculated sensitivity criteria 1 and 2 for misspecifications of the model parameters by 10%, as indicated by the first four columns. The seventh row of this table, for example, evaluates the sensitivity of the $q3$- and the $q5$-plan when the parameter θ_1 deviates from its original by -10%. So, $\bar{\theta}' = (\theta_0 \quad 0.9*\theta_1 \quad \theta_2 \quad \sigma)$.

Row number	Misspecification of model parameters ($\bar{\theta}'$)				CRITERION 1		CRITERION 2	
	θ_0	θ_1	θ_2	σ	$q3$-plan	$q5$-plan	$q3$-plan	$q5$-plan
1	0	0	0	0	1	1	0	0
2	+10%	0	0	0	1.18	1.23	0.00085	0.00096
3	0	+10%	0	0	1.05	1.05	0.00058	0.00038
4	0	0	+10%	0	1.12	1.12	0.00025	0.00018
5	0	0	0	+10%	1.21	1.20	0.000016	0.000018
6	-10%	0	0	0	0.86	0.84	0.00086	0.00095
7	0	-10%	0	0	0.95	0.95	0.00081	0.00053
8	0	0	-10%	0	0.89	0.89	0.00035	0.00024
9	0	0	0	-10%	0.81	0.81	0.000013	0.000015
10	+10%	+10%	+10%	+10%	1.68	1.76	0.0027	0.0023
11	-10%	-10%	-10%	-10%	0.59	0.57	0.0038	0.0034

Table 3.10: Effects of misspecification of the model parameters θ_0, θ_1, θ_2 and σ. The optimum plans were calculated for the values defined in table 2.7.

From this table, the following can be concluded. First, the $q3$-plans are less sensitive to misspecifications of the parameter θ_0. This can be seen from rows 2 and 6: for the $q3$-plan, the values of criterion 1 are closer to 1, while the values of criterion 2 are closer to 0.

The second conclusion is that the $q5$-plans are less sensitive to misspecifications of the model parameters θ_1 and θ_2, which can be concluded from rows 3 and 7 and 4 and 8, respectively. Note that this fact should be underlined, since these two parameters are used for extrapolation to real life conditions.

A last conclusion is that for the model parameter σ, almost no difference in sensitivity between the two plans could be observed.

When studying the sensitivity to misspecifications of each model parameter separately, the two criteria are consistent with each other. Nevertheless, when examining the sensitivity to misspecifications of several model parameters at once, the two criteria are not consistent with each other anymore, as can be observed from rows 10 and 11. A reason for this has not yet been found.

In summary, it can be roughly stated that the difference in sensitivity of the two plans is rather complicated. This result is in disagreement with the literature dealing with the one stress factor case. There, it was clearly observed that plans with more stress levels were less sensitive to misspecifications of the model parameters [MEE84, YAN96]. Such a straightforward conclusion can not be made in the two stress factor case, a result that can not be explained at the moment.

3.4. Forcing the planned experiments to have equal stress times at the two lowest stress levels

When having a closer look at the experimental plans given in tables 2.4 and 2.9, it can be observed that the measurement times of the two lowest stress levels can differ substantially. Sometimes, this might not be convenient from a practical point of view. It might be better to plan experiments for which these measurement times are equal. Such plans make it more easy to practically perform the planned experiment when more than one measurement oven is available.

Forcing the planned experiments to have equal stress times at the two lowest stress levels can be done easily. When minimizing the optimum plan function, OPF, it just comes down to introducing the extra constraint that the stress time of the second stress level is equal to the stress time of the first stress level. In practice, it comes down to forcing the seventh element of the vector $\overline{P}$, introduced in section 2.3.2.2, to be the same as the third element.

The resulting optimal plan having the same input parameters as the $q3$-plan, but now following the extra constraint mentioned above is given in Table 3.11. The EAV's of the MLE's of the parameters θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$, as well as the approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$ are given in Table 3.12.

	Stress levels		
	1	2	3
J (MA/cm²)	0.24	5.50	5.50
ξ_1	0.94	0	0
T (°C)	250.0	161.1	250.0
ξ_1	0	0.43	0
Test time (h)	475.7	475.7	48.6
# DUT's	51	115	133
η (h)	451.1	680.9	20.0
ENF	27.9	56.6	128.3

Table 3.11: Optimum plan of the experiment proposed in this example.

EAV's		**Appr. Conf. Int.**	
Parameter	Value	Parameter	Value
θ_0 (log-hours)	0.0019	η_H (hours)	$20 {}^{-1.7}_{+1.8}$
θ_1	0.0088	n	1.00 ± 0.057
θ_2	0.026	E_a (eV)	0.70 ± 0.030
σ	0.00065	σ	0.50 ± 0.051
$y_N^{0.1\%}$ (log-hours)	0.034	$t_N^{0.1\%}$ (years)	$24 {}^{-7.3}_{+10.6}$

Table 3.12: Numerical evaluation of the experiment proposed in Table 3.11. EAV's of θ_0, θ_1, θ_2, σ and $y_N^{0.1\%}$ and approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$.

From these tables, it can be observed that only small shifts of the stress levels, of the total measurement time per stress level and of the number of DUT's assigned to each stress level occur. The obtained estimated approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and $t_N^{0.1\%}$ do not become much wider. So, the new plan might be preferred to the detriment of the $q3$-plan when it is preferred that the two lowest stress levels have equal measurement times.

3.5. Conclusions

In this chapter, a closer look has been taken at the three major shortcomings of the technique described in chapter 2.

First, a closer look has been taken at the experimental plan that has been calculated in the second example of chapter 2, treated in section 2.4.2. The plan obtained in this section has been referred to as "the $q3$-plan". This plan has been compared with an intuitively planned experiment. It was concluded that the statistical analysis of reliability experiments based on plans that have been designed without a formal method can result in an extremely uncertain estimate of the reliability of the tested product. So, the effort that is done for performing reliability experiments can completely be lost when this experiment is poorly planned.

Then, suggestions were made for forcing the $q3$-plan to have 4 or 5 stress levels. These extra stress levels were forced, for one of the two stress factors, between the lowest and the highest stress level and, for the other stress factor, the extra levels were forced to take the highest possible value. The new optimal plans corresponding to these restrictions have been calculated and the plans were compared with the $q3$-plan. Is was observed that the new plans did not differ substantially from the $q3$-plan with respect to the choice of the stress levels, the choice of the measurement time per stress level and the choice of the proportion of the total amount of DUT's assigned to each stress level. It has also been observed that the width of the approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and t_N^p did not become much wider. When having a closer look at the tables presenting optimal plans with $q \geq 4$, it can be observed that the ENF of the extra stress levels is always equal to the MMNF. In some sense, the plan "does not like" the extra stress levels. The plan assigns "as few as possible" DUT's and "as few as possible" measurement time to the added stress levels. This implies that the parameter MMNF should be set high enough so that the lifetime model can be checked properly.

The sensitivity of the new plans has been compared with the $q3$-plans. Two criteria for comparing these sensitivities have been proposed. It was observed that the difference in sensitivity of the different plans was rather small. A clear-cut conclusion could not be made. When studying the sensitivity to misspecifications of each model parameters separately, the two criteria were consistent with each other. Nevertheless, when examining the sensitivity to misspecifications of several model parameters at once, the two criteria were not consistent with each other anymore. Since the parameters θ_1 and θ_2

are used for extrapolation to real life conditions, is was underlined that the $q5$-plans were less sensitive to misspecification of these parameters. The $q3$-plan, on the other hand, was found to be less sensitive to misspecifications of the parameter θ_0.

Forcing the $q3$-plan to assign the same measurement time to each of the two lowest stress levels turned out to be as easy as forcing the third element of the vector $\overline{P}$, discussed in section 2.3.2.2, to be the same as its seventh element. It was again concluded that the new plan did not differ substantially from the $q3$-plan and that the approximate 95% confidence intervals of the parameters η_H, n, E_a, σ and t_N^p did not become much wider.

4. Comments on the assumptions made by the proposed technique

Concerning the assumptions made by the technique discussed in chapters 2 and 3, two different remarks have to be made. These remarks are given in this chapter and will shortly be enumerated now.

- In chapters 2 and 3, it is assumed that the underlying distribution of failure times is known before planning the experiment. In this chapter, an objective, purely statistical, technique is proposed for finding out whether the failure times coming from a given data set are lognormal or Weibull distributed.
- In previous chapters, it is assumed that the stress values actually applied are equal to the target stress values. In practical situations, this is never the case. The reason for this is discussed in this chapter and its influence on the estimates of the reliability parameters is investigated.

Monte Carlo simulations will be used for approaching the two problems mentioned above. Section 4.1 describes the use of such simulations and explains how reliability experiments should be simulated. Section 4.2 proposes a method for making the distinction between the lognormal and the Weibull distribution. In section 4.3, the influence of the fact that the stress actually applied can be different from the target stress is investigated. In the last section of this chapter, section 4.4, some conclusions are drawn.

4.1. Monte Carlo simulation of reliability experiments

Simulations can be used for obtaining more detailed information on the statistical results of a reliability experiment. The influence of several parameters defining an experiment can be investigated. Simulations allow to "perform" a large number of experiments, such that statistical principles can be better understood, which is not possible while performing a single experiment.

Sampling a failure time from a distribution with distribution function $F(t)$ can be done by following the next procedure [KAL87]:

- Generate a random number p between 0 and 1 using a simple random number generator.

- Find the value t satisfying $F(t) = p$. This value t is the random failure time.

For the Weibull or the lognormal distribution, the distribution function $F(t)$ is given by equation (1.10) or (1.13), respectively. An illustration of this technique is depicted in Figure 4.1. For the lognormal distribution with $\eta = 200$ hours and $\sigma = 0.7$, the random failure time corresponding to a simulated p of, for example, 0.85 is indicated by t_f.

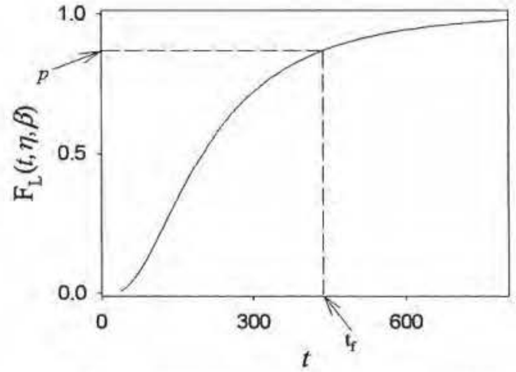

Figure 4.1: Indication of how random failure times are generated using the Monte Carlo technique.

Simulating type I singly censored experiments is now straightforward. The following procedure should be followed.

- Set the underlying distribution function and its dispersion parameter σ. Also set the fitting parameters $\Theta_0, \Theta_1, ..., \Theta_l$ of the acceleration model.
- Set the stress levels $\xi_{i,j}$ ($i = 1, 2, \ldots, l$), the total measurement time, T_j, and the number of DUT's, N_j, per stress level ($j = 1, 2, \ldots, q$).
- For each DUT stressed at the j-th stress level, simulate a failure time using the Monte Carlo technique described above. The median life of the distribution of failure times at this j-th stress level, η_j, can be obtained using the acceleration model and its fitting parameters $\Theta_0, \Theta_1, ..., \Theta_l$. If this simulated failure time is below the total measurement time T_j, the DUT is considered to have failed at its corresponding failure time. If the failure time is higher than the total measurement time, the DUT is considered to be censored at this total measurement time.

In chapter 3, several reliability experiments have been simulated and probability plots of these simulated experiments have been presented.

4.2. Objective method for making the distinction between the lognormal and the Weibull distribution

In this section, a new method for distinguishing between the lognormal and the Weibull distribution is proposed. The method can be applied to both type I and type II singly censored data. Essentially, it comes down to constructing both the lognormal and the Weibull probability plot of the data set under consideration. For each plot, the Pearson's correlation coefficient is calculated. It will be shown that the ratio of these two correlations is a pivotal quantity such that it can be used as a test statistic.

Before starting with a detailed introduction of the problem, some basic statistical definitions will be given. After this, the method will be described in theory and in practice. An illustrative example will be given and a comparison of the new method with the existing methods will be made. At the end, the advantages of the new method will shortly be summarized and conclusions will be drawn.

The method described here has been presented at the ESREF98 conference in Copenhagen in October 1998 [CRO98b]. The method is also incorporated in the software package FAILURE [FAI97].

4.2.1. Basic statistical definitions

A statistical test is generally concerned about the validity of a so-called null hypothesis, H_0. The conclusion of such a test is that this null hypothesis is either or not rejected in favor of the alternative hypothesis H_A. Such test assumes that the truth is either the null hypothesis or the alternative. Testing whether the underlying failure time distribution is lognormal or Weibull can be done using two different combinations of H_0 and H_A: H_0 is lognormal and H_A is Weibull or H_0 is Weibull and H_A is lognormal.

Of course, the conclusion whether the null hypothesis should or shouldn't be rejected can be wrong. Two different parameters dealing with such an error will be discussed now. Both parameters give the probability that the test chooses the alternative hypothesis. The difference between the two is whether the truth is the null hypothesis or the alternative. The first parameter, the significance level α, deals with the case when the null hypothesis is true. α is defined as the probability that the test chooses the alternative hypothesis while the truth is that this null hypothesis is true. The power Π deals with the case when the alternative hypothesis is true. The power of a test is defined as the probability of rejecting the null hypothesis while it should be rejected. It is obvious that a good test has a low significance level and a high power. More detailed definitions of the concepts mentioned above can, amongst others, be found in the work of Freedman et al. [FRE91].

The Pearson correlation coefficient, ρ, of a set of 2 vectors **x** and **y** of length m is defined as

$$\rho = \frac{\sum_{i=1}^{m}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{m}(x_i - \bar{x})^2 \sum_{i=1}^{m}(y_i - \bar{y})^2}}. \tag{4.1}$$

Here, $\bar{x}$ and $\bar{y}$ stand for the mean of the x_i's and the y_i's, respectively. If the correlation coefficient is 0, it implies that there is no linear correlation between **x** and **y**. If the correlation coefficient is 1, it means that the vectors **x** and **y** are perfectly linearly correlated. This fact suggests that a correlation coefficient can serve as a measure for linearity.

4.2.2. Introduction of the problem

Each statistical analysis of reliability data starts with the choice of the underlying distribution of failure times. This choice is of great importance because all conclusions drawn from this analysis will depend on it. The importance of this choice is obvious: the extrapolation to low percentiles is very sensitive to the choice of the underlying distribution of failure times. Today, most researchers choose the underlying distribution of failure times because of "historical" reasons, because everybody uses it,...

For electromigration, for instance, one often assumes a lognormal distribution. A number of papers have been published to support this assumption. One such example is that it is shown that a normal temperature distribution within a sample set produces a lognormal failure rate if the Arrhenius model is assumed [LLO79]. On the other hand, it is shown that a barrier layer can change the failure time distribution of the interconnect electromigration failures [PIN91] and that the length and width of a test stripe can influence the shape parameter β of the underlying distribution of electromigration failures [LAC86]. Another example deals with TDDB experiments. These experiments are mostly fitted to a Weibull distribution since this distribution fits with the weakest-link character of the breakdown process [BAR86; DEG98]. In a paper of Chen et al. [CHE87], however, a lognormal distribution is used. In summary, one can state that some researchers use the lognormal distribution while others use the Weibull distribution in the absence of a theoretical, statistical, justification.

In the literature, only a few statistical methods treating our problem can be found [BAI78]. A method based on the maximum likelihood theory was proposed in a paper of Dumonceaux et al. [DUM73]. A crucial disadvantage of this method is the fact that it is only applicable to data without censoring (so-called complete data). The method as proposed by Kent et al. [KEN82] considered a certain optimal invariant statistic to select the best fitting member of a collection of probability distributions, again using complete samples of life data. A third method, proposed in a paper of Dey [DEY83], attempted to handle complete, interval censored data. A last method handled the problem of type I censoring [SIS82]. It is important to note that all the above mentioned methods, except the one of Dumonceaux et al., are asymptotic methods, meaning that they are only exact when an infinite number of failures are observed. The method as proposed in this thesis is an exact method.

Before introducing the new method, it should be noticed that it is not our intention to overthrow established physical theories concerning the distribution of failure times. Methods established from a physical point of view are of course to be preferred. An example of such method is the one proposed by Lloyd et al. [LLO90] in which the electromigration failure distribution for fine-line interconnects is calculated. Our aim is to provide a statistical tool to distinguish between the lognormal and the Weibull distribution for type I and type II singly censored experiments with an unknown distribution or an unknown failure mechanism.

4.2.3. Method

Since the critical values and the values for the power of the new statistical test are computed using simulated data sets, it will first be explained how singly censored data sets were simulated and which assumptions were made with regard to the censoring times. Then, the statistical quantity of importance will be given and the new method will be explained in theory and in practice.

4.2.3.1. *Simulating singly censored data*

In section 1.3.1, it is stated that for "theoretical" type I or type II singly censored experiments, all unfailed units have a common censoring time and that all failure times are lower than this censoring time. With "practical" singly censored experiments, it might be possible that due to some defect of the measurement equipment, a number of removals occur before the last failure. For simulating such "practical" data sets, assumptions concerning these censored observations have to be made. The specific assumptions we made will be given below.

- Assumptions:
  - The censoring time follows the uniform distribution on the interval $[0,t_f]$, with t_f the failure time of the sample. The density function of the uniform distribution, f_U, on the interval $[a,b]$ is given by:

$$f_U(t;a,b) = \begin{cases} \dfrac{1}{b-a} & \text{if} \quad a<t<b \\ 0 & \text{if} \quad \text{else} \end{cases} \tag{4.2}$$

    A crucial advantage of choosing this uniform distribution is that it does not introduce an extra parameter. A motivation for this choice will be given at the end of this section.
  - Because it is not advisable that the assumption of uniformity has a strong influence on the results presented in this work, it is assumed that the number of observations that are censored before the last failure occurred, defined as cEb, is small with respect to the total number of failures, r. We restricted cEb to be not higher than 10% of r. Note that this assumption is in agreement with practical situations, since the number of censored observations before the last failure is usually far below the total number of failed observations.
  - During the simulations, it is assumed that the experiment is ended after the r-th failure, meaning that all observations which did not fail at the end of the experiment are assumed to be censored at that time (type II censoring). This assumption is of no importance for the proposed method, as will be explained below.

Note that the assumptions mentioned above are only applicable to the observations with a censoring time lower than the failure time of the last failure. The observations that are censored due to stopping the experiment are not subjected to these assumptions. Their censoring times are simply equal to the total duration of the experiment.

- Simulating singly censored data:

It will now be explained how singly censored lognormal or Weibull data sets were simulated. This will be done in 5 successive steps. The final purpose of these steps is to end up with a simulated (lognormal or Weibull) experiment with N devices under test out of which r have failed and cEb were censored before the r-th failure. Figure 4.2 illustrates the method in a graphical way.

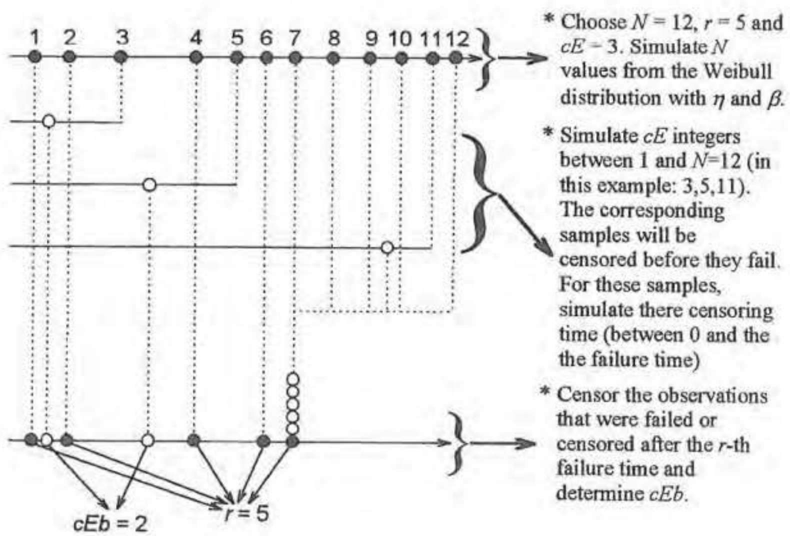

Figure 4.2: Example of how a multiple censored data set is simulated. The black circles at the upper line are the N simulated Weibull failure times. The white circles on the three lines below the upper line are the censoring times of the cE samples that are censored before they fail. The lowest line is the final data set. It can now be determined that cEb = 2.

- Specify N, r and cE. cE is defined as the number of censored components in case the experiment is not stopped until the time that all components have left the experiment due to failure of whatever other reason. In mathematical terms: $cE = N\text{-}r(t = \infty)$. Another definition can be: cE is number of components that will be censored before they fail. Note that cE is not known in real experiments.

- Generate, using the Monte-Carlo technique, N lognormal or Weibull observations with parameters η and β.
- Choose at random cE components that will be censored.
- Generate the censoring times of these components (uniform on the interval [0,actual failure time]).
- Set the censoring time of the observations that were failed or censored after the r-th failure time equal to this r-th failure time and determine cEb.

In the rest of this section, the choice of the uniform distribution as a model for the distribution function of censoring times will be motivated. The exponential distribution with some scale parameter η would be a logical choice because this is a frequently used distribution in the medical world for modeling censored observations. The popularity of the exponential distribution for this purpose comes from the fact that it is a so-called memoryless distribution (constant hazard rate). However, because a lot of reasons exist for a sample to be censored, the scale parameter η would depend on an enormous number of different factors, even on the measurement equipment itself. So, because a generally applicable estimate of η does not exist, it is practically impossible to use the exponential distribution for our purpose.

The uniform distribution has been chosen from the viewpoint of the simulations. Suppose that we know the failure time of a sample and that we also know that this sample is going to be censored before it fails. In that case, a constant hazard rate is not advisable anymore and the uniform distribution, which has a linearly increasing hazard rate, is an obvious alternative.

Although is has been tried here to found the assumption of uniformity as good as possible, its validity is hard to verify since in most practical situations only few observations are censored before the last failure.

4.2.3.2. Statistical quantity of importance

In order to be able to calculate the statistical quantity of importance for a given data set, it is necessary to create both the Weibull and the lognormal probability plot of this data set. For details with regard to creating probability plots, we refer to the standard work of Nelson [NEL90].

For the given data set, ρ_W is defined as the Pearson correlation coefficient of the points on its Weibull plot, while ρ_L is defined as the Pearson correlation coefficient of the points on its lognormal plot. The statistical quantity of importance is the ratio ρ_W/ρ_L.

It will now be shown that the distribution of the ratio ρ_W/ρ_L over different experiments only depends on N, r, cEb and on the underlying failure distribution. So, the distribution of the ratio ρ_W/ρ_L does NOT depend on the median life η and on the dispersion parameter σ of the underlying failure distribution, hence it is a pivotal quantity. A mathematical definition of the concept of a pivotal quantity can be found in appendix A.

Figures 4.3 and 4.4 clarify the statement of ρ_W/ρ_L being a pivotal quantity. These figures show the result of calculations performed on 160,000 simulated data sets with $N = 60$, $r = 40$ and $cEb = 2$. The underlying failure time distribution of these data sets was chosen to be lognormal. The first 80,000 data sets were simulated with $\eta = 200$ hours and $\sigma = 2$ as parameters for the underlying lognormal distribution, while for the other 80,000 data sets, $\eta = 17$ hours and $\sigma = 0.7$ was chosen. For each data set, the ratio ρ_W/ρ_L was calculated. The full line on Figure 4.3 is the estimated density function of these ρ_W/ρ_L for the first choice of η and σ, while the dotted line is the estimated density function of these ρ_W/ρ_L for the second choice of η and σ. The lines depicted in Figure 4.4 show the estimated distribution functions for both choices of η and σ. On the face of it, it can be observed that the distribution of ρ_W/ρ_L does not depend on these η and σ. In this work, an objective criterion has been used for deciding whether two distributions were equal or not: when the maximal absolute difference with respect to the x-axis of two estimated distribution functions was smaller than 0.0005, the two distributions corresponding to these estimated distribution functions were considered to be equal. The choice of this specific value 0.0005 will be explained in section 4.2.3.4. Note that far more accepted rules for deciding whether two distributions are equal or not are goodness-of-fit tests, like for example the Kolmogorov-Smirnov test [BIR94]. However, such tests reject the null hypothesis of the two distributions to be equal also when there is only a small difference between the two. In this work, we are not interested in such small differences, making goodness-of-fit tests not useful.

More plots like those depicted below were inspected for more combinations of N, r and cEb, of the underlying failure distribution and of η and σ. Inspection of all plots resulted in the same conclusion: ρ_W/ρ_L is, to a good approximation, not a function of the median life η and the dispersion parameter σ and hence it is a pivotal quantity. By using the objective criterion mentioned above, it has been verified that the property of ρ_W/ρ_L being a pivotal quantity remains valid when *cEb* is lower than around 30% of *r*. Remember that in this work *cEb* has been restricted to be lower than 10% of *r*.

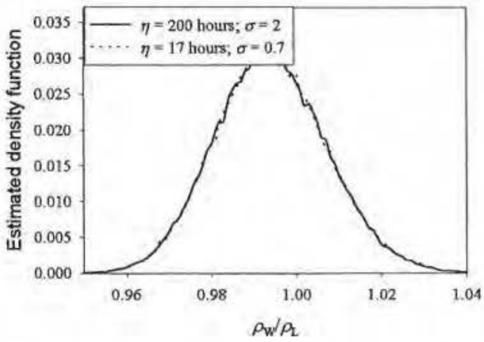

Figure 4.3: Estimated density functions of ρ_W/ρ_L with N=60, r=40 and cEb=2. The underlying failure distribution is lognormal. The difference between the two curves is due to the choice of the values for η and σ.

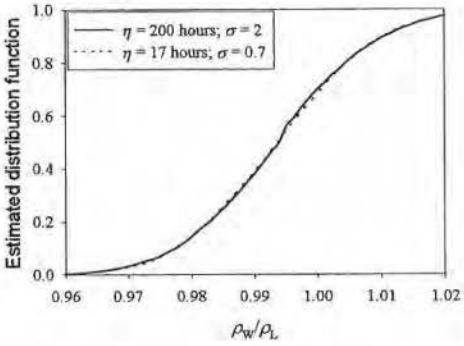

Figure 4.4: Estimated distribution functions of ρ_W/ρ_L with N=60, r=40 and cEb=2. The underlying failure distribution is lognormal. The difference between the two curves is due to the choice of the values for η and σ.

4.2.3.3. Method in theory

Figure 4.5 shows the estimated density functions of the calculated ρ_W/ρ_L values of 160,000 simulated data sets, 80,000 for each curve, with N = 60, r = 40 and cEb = 2. The difference between the two curves is in the choice of the underlying failure distribution (from previous section, it is known that the choice of η and σ is of no importance). The full line corresponds to the simulations with an underlying lognormal distribution, while the dotted line represents the simulations with an underlying Weibull distribution. Define the estimated density curve of ρ_W/ρ_L for Weibull samples as $DE_W(\rho_W/\rho_L)$ and the estimated density curve of ρ_W/ρ_L for lognormal samples as $DE_L(\rho_W/\rho_L)$.

It can be observed from the figure that ρ_W/ρ_L tends to be higher if the underlying failure distribution is Weibull. This is to be expected: Weibull samples give a higher correlation for the Weibull plot where lognormal samples give the opposite.

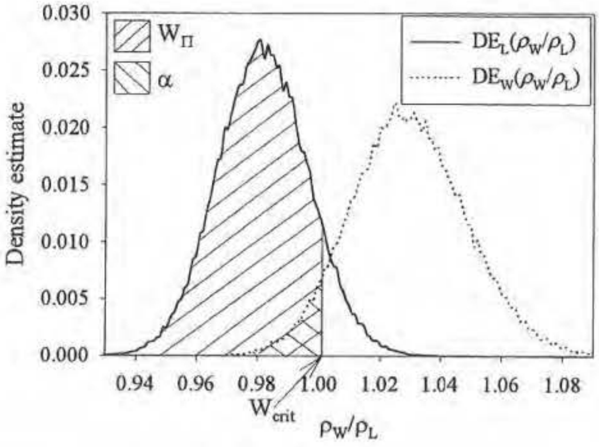

Figure 4.5: Two plots of the estimated density functions of ρ_W/ρ_L for data sets with $N = 60$, $r = 40$ and $cEb = 2$. The difference between the two curves is the underlying distribution function of failure times. This figure explains how to test H_0: Weibull versus H_A: Lognormal. The values α and W_Π are defined as the shaded areas below the curves.

Testing the null hypothesis H_0: Weibull versus the alternative H_A: lognormal can most easily be done using the technique depicted in Figure 4.5. The next 4 steps should be followed:

- Set the significance level α of the test. Typical values for α are 0.05 or 0.1.
- Find the critical value W_{crit} of the test. This is the value on the x-axis which divides $DE_W(\rho_W/\rho_L)$ into two parts: the left part containing α * 100% of the total area under the curve. So, W_{crit} fulfills the following equation:

$$\alpha = \frac{\int_{-\infty}^{W_{crit}} DE_W(\rho_W/\rho_L)\, d(\rho_W/\rho_L)}{\int_{-\infty}^{\infty} DE_W(\rho_W/\rho_L)\, d(\rho_W/\rho_L)} \tag{4.3}$$

- For a given data set, the hypothesis H_0: Weibull is rejected when the observed ratio ρ_W/ρ_L is lower than W_{crit}. The probability to observe such ratio ρ_W/ρ_L which is lower than W_{crit}, given that H_0 is true, is exactly α.
- The power Π of this test is defined as W_Π. W_Π is the area under the estimated density curve of ρ_W/ρ_L for lognormal samples for the x-values below W_{crit}. So,

$$W_\Pi = \frac{\int_{-\infty}^{W_{crit}} DE_L(\rho_W/\rho_L)\,d(\rho_W/\rho_L)}{\int_{-\infty}^{\infty} DE_L(\rho_W/\rho_L)\,d(\rho_W/\rho_L)} \tag{4.4}$$

The test H_0: Lognormal versus H_A: Weibull can be performed using a similar technique as presented above. The critical value of this test, L_{crit}, is defined as the value which fulfills the following equation:

$$\alpha = \frac{\int_{L_{crit}}^{\infty} DE_L(\rho_W/\rho_L)\,d(\rho_W/\rho_L)}{\int_{-\infty}^{\infty} DE_L(\rho_W/\rho_L)\,d(\rho_W/\rho_L)}. \tag{4.5}$$

H_0: Lognormal is rejected when the observed ρ_W/ρ_L is higher than L_{crit}. L_Π defines the power of this test. We have:

$$L_\Pi = \frac{\int_{L_{crit}}^{\infty} DE_W(\rho_W/\rho_L)\,d(\rho_W/\rho_L)}{\int_{-\infty}^{\infty} DE_W(\rho_W/\rho_L)\,d(\rho_W/\rho_L)}. \tag{4.6}$$

It should be noted that the proposed method implies the assumption that the data are monomodal distributed. On top of that, it is assumed that the data follow either the lognormal or the Weibull distribution. Of course, it is always possible that the true underlying distribution does not meet with these assumptions. In that case, the method can be used for finding out which of the two distributions, lognormal or Weibull, is the best approximation for the true underlying distribution function.

Also note that because censored observations are not plotted on a probability plot, it is of no importance whether the experiment is type I or type II singly censored.

It is also important to note that the proposed method can only be used for continuous monitoring. Nevertheless, it can serve as an approximate method for interval monitoring. Of course, the more narrow the intervals, the better the approximation. No simulations have been performed in order to check the degree of appropriateness when the method was applied to interval censored data. At our institute, the degradation of components is measured with a very high frequency with respect to the total measurement time, such that the assumption of continuous monitoring is justified.

4.2.3.4. Method in practice

From the fact that the ratio ρ_W/ρ_L is a pivotal quantity, it follows that the values W_{crit}, W_{Π}, L_{crit} and L_{Π} only depend on α, N, r and cEb. In this thesis, these critical values and the values for the power were calculated for $N = 10, ..., 128$; $r = 10, ..., N$; $cEb \leq 0.1 * r$, and $\alpha = 0.4$, 0.2, 0.1 and 0.05. This is done by simulating 80,000 samples for each combination of N, r, cEb and for the two underlying failure distributions under consideration. The critical values and the values for the power were obtained using the method presented in the previous section. The accuracy of the calculated values for the power was higher than 0.02, where for the critical values, an accuracy higher than 0.002 was attained.

The rule of thumb for assuming two distributions to be equal in section 4.2.3.2 is verified by this last accuracy. This will be explained now. Remember that in section 4.2.3.2 two distributions were considered to be equal when the maximal shift with respect to the x-axis of the two estimated distribution functions was less than 0.0005. This specific value 0.0005 is 4 times less than the obtained accuracy of 0.002 on the calculated critical values of our test. This guarantees that the maximal shift of the distribution of ρ_W/ρ_L, caused by the fact that ρ_W/ρ_L is not pivotal, is at least a factor 4 smaller than the accuracy on the calculated critical values, which is a safe margin.

Figure 4.6 shows the values of the power of the two tests H_0: Weibull and H_0: Lognormal for varying r and constant N (= 64) and cEb (= 0). α is chosen to be 0.05. It can be observed that the power increases if the number of failures increases. This can be explained by the fact that more failures reveal more information about the underlying failure distribution.

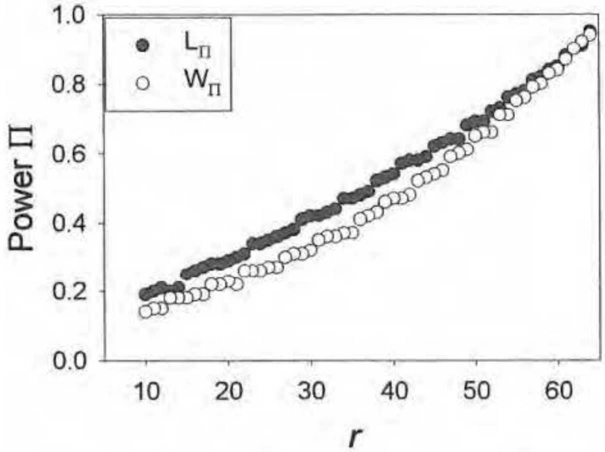

Figure 4.6: Plot of L_Π *and* W_Π *for varying r.*
$N = 64$, $cEb = 0$ and $\alpha = 0.05$.

Figure 4.7 shows the value of the power for constant r (= 20) and cEb (= 2) as a function of N. It can be observed that the power decreases for increasing N. At first sight, this is surprising, but it can be explained as follows: increasing N with constant r implies a smaller range covered by the y-axis of a probability plot, such that the information with respect to the y-axis decreases. This is illustrated in Figure 4.8, which shows the lognormal probability plot of two simulated data sets with r (= 20) and cEb (= 2). One data set was simulated using $N = 20$, while for the other data set, N was set to 60. As can be observed, the range covered by the y-axis is largest for the first data set.

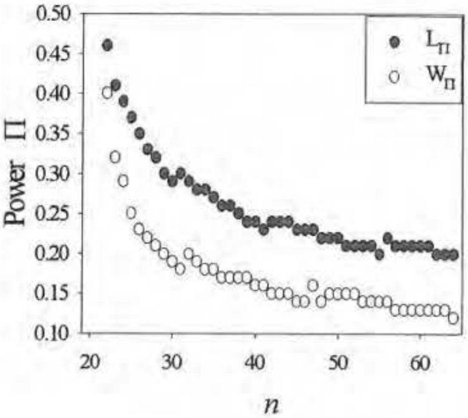

Figure 4.7: Plot of L_Π *and* W_Π *for varying* N. $r = 20$, $cEb = 2$ *and* $\alpha = 0$.

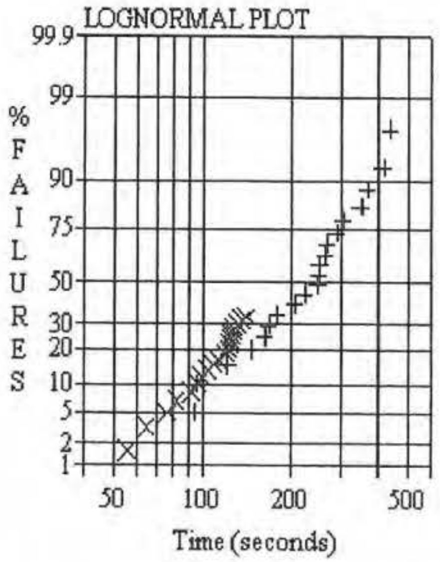

Figure 4.8: Lognormal probability plot of two simulated data sets with r *(= 20) and* cEb *(= 0). One data set (+) was simulated using* $N = 20$*, while for the other data set (X),* N *was set to 60.*

Figure 4.9 shows the power of both tests for constant N (= 60) and r (= 50), but for varying cEb. Observe that the power increases with increasing cEb. This can again be explained by the fact that the range covered by the y-axis gets wider with increasing cEb. Observe that in Figure 4.9, L_Π increases from about 62% to 69%, which is a rather large power increase.

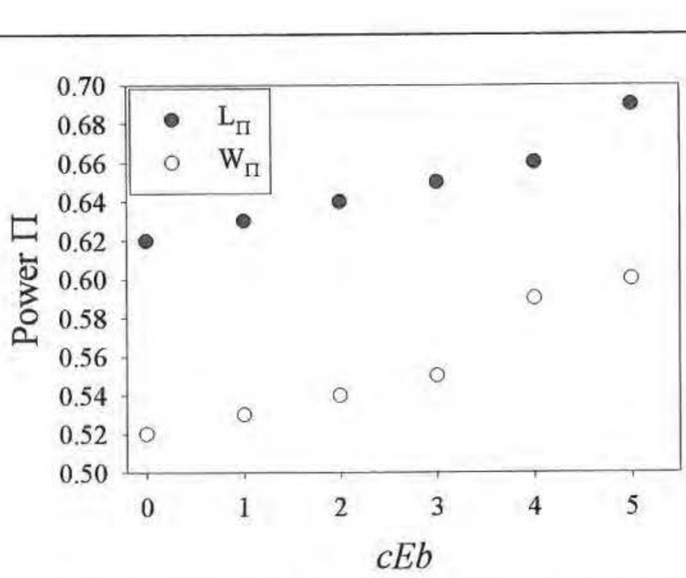

Figure 4.9: Plot of L_Π *and* W_Π *for varying cEb.*
$N = 60$, $r = 50$ *and* $\alpha = 0.05$

4.2.4. Illustrative example

Figure 4.10 shows both the lognormal and the Weibull probability plot of a data set with the following characteristics: $N = 40$, $r = 40$ and $cEb = 0$. The 40 failure times have been simulated from either the lognormal or the Weibull distribution. Without a formal method, it is hard to see which distribution fits the data in the best way. In this example, our test will be used for finding out which distribution these data have been simulated from.

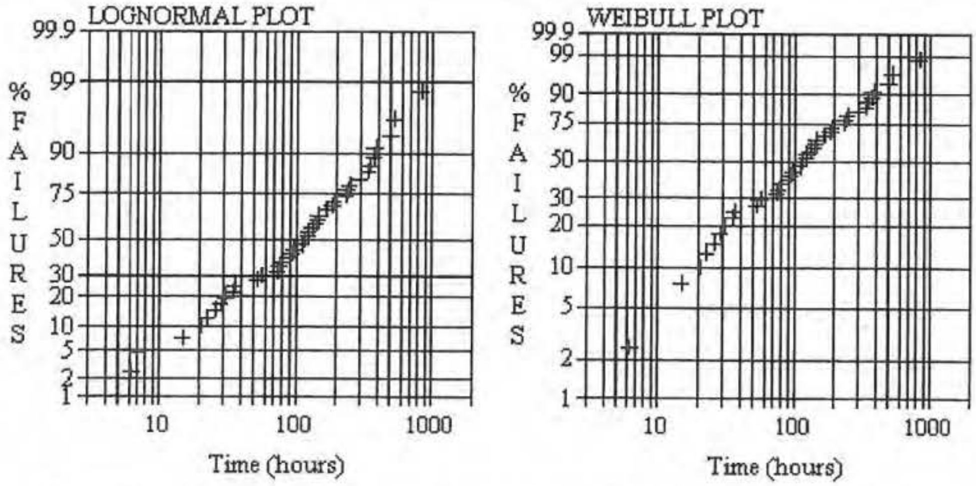

Figure 4.10: Lognormal and Weibull probability plot of 40 simulated failure times

Table 4.1 gives the critical values and the values of the power for testing H_0: Weibull and H_0: Lognormal for several values of α for the specific case of $N = 40$, $r = 40$ and $cEb = 0$.

	H_0: Weibull		H_0: Lognormal	
α	W_{crit}	W_{Π}	L_{crit}	L_{Π}
0.05	0.993	0.69	1.014	0.73
0.1	1.000	0.84	1.007	0.83
0.2	1.009	0.92	0.999	0.91
0.4	1.022	0.98	0.989	0.97

Table 4.1: Summary of testing H_0: Weibull and H_0: Lognormal for the example considered in the text

If one wants to find out whether to choose the lognormal or the Weibull distribution, without having a prior belief in one of the two distribution functions, testing H_0: Weibull or H_0: Lognormal at an α-level such that $\alpha \approx 1\text{-}\Pi$ is advised. With this specific choice of α and Π, the probability of making the wrong *decision* does not depend on the *truth*. So, no distribution function is favored with respect to the other.

Let us, for example, test H_0: Weibull at an α-level of 0.2. This means that if we reject the null hypothesis, the probability of making the wrong decision is equal to 0.2. If we perform the test and we find that the null hypothesis should not be rejected, the probability that this result is wrong equals 1-0.92=0.08. We see that the critical value of the test equals 1.009. So, if the observed value for ρ_W/ρ_L is lower than 1.009, we can decide to reject the null hypothesis.

We have now calculated that the observed value for ρ_W/ρ_L equals 1.011, which is higher than 1.009, so we don't reject H_0: Weibull. So, it is decided to chose the Weibull distribution and it is known that the probability of making the wrong decision is only 0.08. And indeed, the underlying distribution function these data were simulated form was…Weibull.

From Table 4.1, it has to be noted that the values for the power of these tests are relatively high. This will not be the case for a lower number of failures. So, for finding the correct failure time distribution, a significantly high number of failures is mandatory.

The danger of choosing the wrong distribution function can best be seen while comparing the results of a lognormal and a Weibull fit. The estimated 0.01%-percentile for the lognormal fit performed on the failure times of Figure 4.10 was 1.41 hours, while this estimate amounted to 0.0218 hours for a Weibull fit. This is a ratio of 65!

4.2.5. Comparison with the literature

The values for the power of the tests H_0: Weibull and H_0: Lognormal calculated in this thesis are compared with those obtained in the literature. This is done for methods treating complete data sets and for methods treating type I singly censored data sets. As mentioned in section 4.2.1, methods considering the other types of data sets treated in this thesis could not be found in the literature.

4.2.5.1. Complete data sets

For complete data sets with $N = 30$, $r = 30$ and $cEb = 0$, Table 4.2 gives the values for the power of the tests proposed in this thesis and of the tests proposed in Dumonceaux et al. [DUM73] and Siswadi et al. [SIS82]. The values for the power of the tests corresponding to an "x" were not given in the paper under consideration.

	H_0: Weibull			H_0: Lognormal		
α	This method	Dumonceaux	Siswadi	This method	Dumonceaux	Siswadi
0.05	0.56	0.62	x	0.62	0.63	x
0.1	0.71	0.74	x	0.73	0.75	± 0.74
0.2	0.84	0.84	± 0.84	0.84	0.86	± 0.86

Table 4.2: Values for the power of the test H_0: Weibull and H_0: Lognormal for $N = 30$, $r = 30$ and $cEb = 0$ proposed in this work and of the tests proposed in Dumonceaux et al. [DUM73] and Siswadi et al. [SIS82].

As can be observed, the values for the power of the tests proposed in Dumonceaux et al. are slightly higher than those of the tests proposed in this thesis. The powers of the tests proposed in Siswadi et al. are more or less the same as those proposed in this thesis. From this, it can be concluded that, for complete data sets, our new method behaves as well as those described in the literature.

4.2.5.2. Type I singly censored data sets

Table 4.3 gives the values for the power of the tests proposed in this thesis and in Siswadi et al. [SIS82] for data sets with $N = 30$, $r = \pm\ 27$ and $cEb = 0$.

	H_0: Weibull		H_0: Lognormal	
α	This method	Siswadi	This method	Siswadi
0.2	0.70	± 0.58	0.71	x
0.4	0.88	x	0.86	± 0.80

Table 4.3: Values for the power of the tests H_0: Weibull and H_0: Lognormal for $N = 30$, $r = 27$ and $cEb = 0$ proposed in this thesis and of the tests proposed in Siswadi et al. [SIS82].

As can be observed, the values for the power of the test proposed in this thesis are higher than those of the test proposed in Siswadi et al., such that it can be concluded that for type I singly censored experiments, the new method is better than those described in the literature.

4.2.6. Conclusions and advantages of the new method

In this section, a formal method has been proposed for making the distinction between the lognormal and the Weibull distribution. This method was based on the ratio ρ_W/ρ_L. It has been shown that this ratio was pivotal in practical situations were the number of observations that were censored before the last failure was small. Based on the ratio ρ_W/ρ_L, a test was proposed and the critical values and the values for the power of this test were calculated based on Monte-Carlo simulations. An illustrative example has been given and the method was compared to those described in the literature. We found that our method had a higher power than those obtained in the literature, except for the, in practical situations seldom found, case of complete data.

The first and foremost advantage of the new method is that it is easy-to-use. The method is simply based on the principle of probability plotting.

A second advantage deals with the power of the tests. For the proposed test, the power is always known, such that the probability of making the wrong decision can be controlled by the user.

All methods described in the literature are, except the one of Dumonceaux et al. [DUM73], asymptotic methods, meaning that they are only exact when an infinite number of failures are observed. The method as proposed in this thesis is an exact method.

The last advantage of the proposed method is that the DUT's that were censored before the last failure are taken into account.

4.3. The influence of gradients in the applied stress

In reliability experiments, it can occur that the stress actually applied differs from the target stress. When applying temperature stress, for example, a large number of DUT's is put in one oven. The temperature in that oven is not the same at all positions. In the lower part of the oven, the temperature is generally lower than in its upper part. Another example deals with current stress. Applying the same current to lines with slightly different line widths results in slightly different current densities. A last example comes from the world of testing the reliability of MOSFET's. Applying the same drain voltage V_d and the same gate voltage V_g to several MOSFET's having a slightly different channel length will result in slightly different channel currents. The difference between the actually applied stress and the desired stress is often referred to as stress gradients.

Some researchers perform corrections for such gradients [SCO97] because they believe that this will improve the quality of their reliability experiment. Is such expensive and time-consuming work really necessary? In this section, results of simulations that have been performed in order to study this problem are presented.

The major purpose of these simulations is to answer the following question: "Does a stress gradient have an influence on the estimates of the model parameters Θ_0, Θ_1, ..., Θ_l and on β?"
After these investigations, the following crucial question will be answered: "If there are influences, how do they translate to the estimates of low percentiles at real life conditions?"

4.3.1. Introduction of the simulations

4.1.1.1. Parameters whose influence has been investigated

The influence of oven gradients is investigated for temperature storage experiments. The lifetime model is assumed to be the Arrhenius relation. Experiments with only one stress temperature ($q = 1$) and experiments with three stress temperatures ($q = 3$) are considered. For the experiments with one stress level, the stress temperature is set to 230 °C. The median life η of the underlying distribution function of failure times is chosen to be 200 hours. For the experiments with three stress levels, the stress temperatures are set to 200 °C, 230 °C and 260 °C. Again, the median life η at 230 °C is chosen to be 200 hours.

Three values for the dispersion parameter σ of the underlying distribution function of failure times are assumed: 0.5, 1 and 1.5 for experiments with lognormal distributed failure times and 2, 0.5 and 0.2 for experiments with Weibull distributed failure times.

Two values for the activation energy E_a are assumed: 0.7 and 1.5 eV.

Type II singly censored experiments are simulated. The number of DUT's per stress level, N_j, is chosen to be 16, 32 or 64. When q is set to 3, these values are taken to be the same for each stress level. Data sets with two different "degrees" of censoring are considered. First, data sets having no censored observations ($r_j = N_j$, $j = 1, \ldots, q$) are simulated. These data sets will be referred to as "not censored". Second, experiments having $r_j = N_j/2$ for $j = 1, \ldots, q$ are simulated. These data sets will be referred to as "50% censored".

Three different temperature profiles in the oven are considered. Define T_{grad} as the maximal temperature difference between two oven positions. Normally, T_{grad} is not higher than 5 °C. The simulations are performed for values of T_{grad} = 0, 2, 4, 6, 8, 10, 12, 14, 16 °C. The three different temperature profiles that have been considered in this work are described now.

- Temperature profile 1:
  The temperature in the oven is Normal distributed with mean equal to T_{target} and a variance equal to $(T_{grad}/6)^2$. Assuming this variance, is it is guaranteed that approximately 99% of the oven positions have a temperature between $T_{target} \pm T_{grad}/2$.

- Temperature profile 2:
  The temperature is distributed according to the random variable with density function $f_2 = (f_X + f_Y)/2$. Here, f_X is the density function of a normally distributed random variable X with mean $T_{target} + T_{grad}/6$ and variance $(T_{grad}/6)^2/4$. f_Y is the density function of a normally distributed random variable with mean $T_{target} - T_{grad}/6$ and variance $5*(T_{grad}/6)^2/4$. The variable corresponding to density function f_2 has mean T_{target} and variance $(T_{grad}/6)^2$. Observe that these values are the same as those of the distribution assumed in profile 1.

- Temperature profile 3:
  The temperature is distributed according to the random variable with density function $f_3 = (f_X + f_Y)/2$. Here, f_X is the density function of a normally distributed random variable X with mean $T_{target} - T_{grad}/6$ and standard deviation equal to $(T_{grad}/6)^2/4$. f_Y is the density function of a normally distributed random variable with mean $T_{target} + T_{grad}/6$ and variance $5*(T_{grad}/6)^2/4$. The random variable corresponding to density function f_3 again has mean T_{target} and variance $(T_{grad}/6)^2$.

Temperature profiles 2 and 3 are asymmetric. These have been introduced because the shape of a temperature profile in an oven is not known. Figure 4.11 shows examples of the three temperature profiles described above for T_{target} = 200°C and T_{grad} = 10°C.

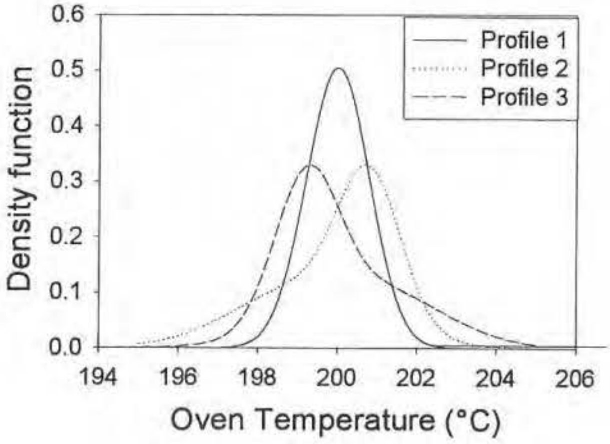

Figure 4.11: Stress profiles for $T_{target} = 200°C$ and $T_{grad} = 10°C$

A summary of all parameters whose influence with respect to temperature gradient effects is investigated is given in Table 4.4.

Parameter	**Possible values**
q	1 or 3
Distribution	Lognormal or Weibull
σ	0.5, 1 or 1.5 if lognormal 2, 0.5 or 0.2 if Weibull
E_a (eV)	0.7 or 1.5 eV
N_j	16, 32 or 64 for $j = 1, \ldots, q$
Censoring	"Not censored" or "50 % censored"
T_{grad} (°C)	0, 2, 4, 6, 8, 10, 12, 14 or 16
Temperature profile	1, 2 or 3

Table 4.4: Summary of all parameters whose influence with respect to temperature gradient effects has been investigated.

4.3.1.2. Details and major purposes of the simulations

For each combination of the parameters enumerated in Table 4.4, 10,000 experiments are simulated. The MLE of η, E_a and σ is obtained for each simulated experiment. Then, the influence of T_{grad} on the median of the 10,000 MLE's of the parameters η, E_a and σ are investigated. The result of these investigations is given in the next three sections.

4.3.2. Influence on the estimates of the median life η

It will be investigated how temperature gradients in a measurement oven can influence the estimate of the median life η. Remember that such an estimate is obtained when statistically analyzing the data that are measured during a reliability experiment. The change of the final estimate of η caused by the introduction of a temperature gradient will be studied in this section.

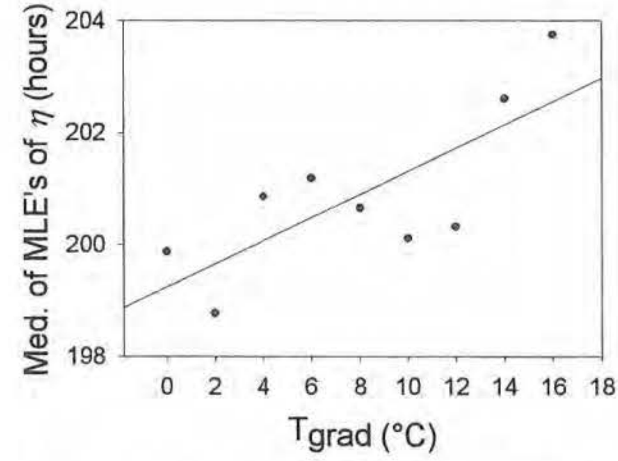

Figure 4.12: Median of the MLE's of η of 10,000 simulated experiments as a function of T_{grad}. The input parameters of the simulated experiments are enumerated in the text.

Figure 4.12 shows the median of the MLE's of η of 10,000 simulated experiments as a function of T_{grad}. The experiments are simulated using the following input parameters: $q = 3$, underlying lognormal distribution, $\sigma = 1.5$, $E_a = 0.7$eV, $N_j = 32$, 50% censored and temperature profile 1. A linear influence of T_{grad} on the median of the estimated η's can be observed. The points on this plot were fitted to a straight line: $a_\eta + b_\eta$*T_{grad}. The slope of this line, b_η, was estimated using simple linear regression. The estimate of b_η, $\hat{b}_\eta$, of this line amounts to 0.21 hours/°C.

Plots like these were made for all combinations of the parameters discussed in Table 4.4 and the medians of the MLE's of η were fitted to a straight line. The slopes b_η have been estimated and a 99% confidence interval was calculated. From these calculations, the following conclusions can be drawn:

- From all $\hat{b}_\eta$, about half were found to be significantly different from zero. All significant $\hat{b}_\eta$'s were positive.
- Increasing the activation energy E_a from 0.7 to 1.5eV leads to an increase of the significant $\hat{b}_\eta$'s.
- Going from experiments with no censoring to experiments with 50% censoring caused a significant increase on the significant $\hat{b}_\eta$'s of experiments with an underlying lognormal distribution. This effect was much smaller for experiments with an underlying Weibull distribution.
- The choice of the temperature profile, the choice of N_j as well as the choice of q almost had no influence on the slopes.

The fact that about half of all estimated slopes $\hat{b}_\eta$ were significantly different from zero is counterintuitive. If T_{grad} has an effect on the median of the MLE's of η, one would expect that almost all $\hat{b}_\eta$'s are significantly different from zero. However, from Figure 4.12, it can be observed that the variance of the obtained medians is rather large. So, significant $\hat{b}_\eta$'s will only be obtained when this variance is smaller than the effect of increasing medians. The variance of the obtained medians can be reduced by increasing the number of simulations.

Finding explicit relations between the parameters of Table 4.4 and the estimated slopes $\hat{b}_\eta$ is difficult because the number of levels chosen for the parameters of Table 4.4, was at maximum 3 (for N_j and σ). Finding explicit relations between these parameters and the estimated slopes $\hat{b}_\eta$ will only be possible when this number of levels is increased.

The fourth column of Table 4.5 on page 117 gives an impression of the order of magnitude of the significant slopes $\hat{b}_\eta$. This column gives the highest significant $\hat{b}_\eta$'s as a function of the underlying distribution, of the activation energy E_a and of the dispersion parameter σ. So, among all combinations of q, N_j, stress profile and degree of censoring, the highest significant slope is given.

As can be observed, the highest estimated slope amounts to 0.55 hour/°C. So, suppose that one performs a temperature storage experiment with a measurement oven at 230°C and one knows that the median lifetime of the DUT's is 200 hours. When the temperature gradient of the measurement oven used in the experiment is about 5°C, it can be concluded that the estimated median lifetime can maximally shift from 200 to 203 hours, which is 1.5%. This is, in practical situations, a negligible result.

It is important to note that it has been simulated that this result is proportional to the true value of the median life η. So, when assuming η to be, say, 20 hours, a maximal shift of around 0.055 hour/°C was found.

4.3.3. Influence on the estimates of the activation energy E_a

In this section, it will be investigated how temperature gradients can influence the final estimate of the activation energy E_a. The change of the MLE's of the activation energies caused by the introduction of a temperature gradient will be studied.

Plots like the one depicted in Figure 4.12 have been made and a linear fit has been performed. The slopes of these lines, b_{E_a}, have been estimated and a 99% confidence interval was calculated for all combinations of the parameters discussed in Table 4.4. From these calculations, it can be concluded that no significant $\hat{b}_{E_a}$'s could be found. So, T_{grad} has no influence on the estimated activation energy.

4.3.4. Influence on the estimates of the dispersion parameter σ

In this section, the dependence of a temperature gradient on the final estimate of the dispersion parameter σ will be studied using the same procedure as in the previous two sections. For the lognormal distributed failure times, the slopes b_σ of the linear relation between T_{grad} and the median of the MLE's of the dispersion parameter σ have been calculated for all different combinations of the parameters discussed in Table 4.4 and 99% confidence intervals of these estimated slopes $\hat{b}_\sigma$ have been calculated as well.

For Weibull distributed failure times, we have calculated the effect on the shape parameter β, because the relation between the median of the estimated σ's and T_{grad} could not be assumed to be linear. The relation between T_{grad} and the estimated β's, however, could be assumed to be linear. So, for Weibull distributed failure times, the slopes b_β have been estimated and a 99% confidence interval has been calculated.

From these results, the following conclusions can be drawn:

- From all $\hat{b}_\sigma$'s and $\hat{b}_\beta$, about 70% were found to be significantly different from zero. For experiments with an underlying lognormal distribution, the $\hat{b}_\sigma$'s were positive, while for experiments with Weibull distributed failure times the $\hat{b}_\beta$ were all negative. So, a temperature gradient leads to an increase of the dispersion parameter σ.
- Increasing the activation energy E_a from 0.7 to 1.5eV led to an increase of the absolute value of the significant $\hat{b}$'s.
- For experiments with Weibull distributed failure times, it could be observed that when decreasing the dispersion parameter σ, the absolute value of the significant $\hat{b}_\beta$'s increased. For experiments with lognormal distributed failure times, such an effect could not be observed.
- Going from experiments with no censoring to experiments with 50% censoring caused a large increase of the significant $\hat{b}_\sigma$'s for experiments with an underlying lognormal distribution. There was not such an effect for experiments with an underlying Weibull distribution.
- A slight increase of the absolute value of the significant $\hat{b}$'s could be observed when increasing N_j or q.
- The choice of the temperature profile had no influence on the significant slopes.

The fifth column of Table 4.5 gives an impression of the order of magnitude of the significant slopes $\hat{b}$. It can be seen that the largest shift in terms of percentage of the dispersion parameter σ occurred when the underlying distribution of failure times is Weibull and when $\sigma = 0.2$ and $E_a = 1.5$eV. In that particular case, β shifts from 5 to 4.45 when $T_{grad} = 5$°C.

The investigation of the practical consequences of such a shift is not so straightforward. This can best be done by investigating its influence on the estimation of low percentiles under real life conditions. This will, amongst others, be done in the next section.

4.3.5. Influence on the estimates of low percentiles

The results from the previous three sections will now be translated to the problem of estimating low percentiles at real life conditions. More concrete, it will be investigated how the shifts of the model parameters η and σ influence these low percentiled.

From equations (2.6) and (2.16), it can be calculated that for temperature storage experiments for which the Arrhenius model is true, the p%-percentile under real life conditions, t_N^p, is given by:

$$t_N^p(\eta_{\mathrm{T}}, E_a, \sigma) = \eta_{\mathrm{T}} * \exp\left[\frac{E_a}{\mathrm{k_B}}\left(\frac{1}{\mathrm{T}_N} - \frac{1}{\mathrm{T}}\right)\right] * \exp\left(\sigma * \mathrm{F_N^{-1}}(p)\right) \tag{4.7}$$

Here, T_N is the real life temperature and η_{T} is the median of the distribution of failure times at some temperature T. Remember from equation (2.16) that for experiments with lognormal distributed failure times, $\mathrm{F_N^{-1}}(p)$ is defined as the p%-percentile of the standard Normal distribution function. For experiments with Weibull distributed failure times, $\mathrm{F_N^{-1}}(p)$ is defined as the p%-percentile of the standard extreme value distribution: $\mathrm{F_N^{-1}}(p) = \ln(-\ln(1 - p/100))$. It is important to observe that this $\mathrm{F_N^{-1}}(p)$ is negative when p is smaller than 0.5 for lognormal distribution. For Weibull distributions, $\mathrm{F_N^{-1}}(p)$ is negative when p is smaller than 0.63. Since the major interest of reliability experiments goes to the estimation of low percentiles, $\mathrm{F_N^{-1}}(p)$ will be considered to be negative throughout the rest of this section.

Remember that for the simulations presented here, it is assumed that, at T = 230°C, the median life η amounts to 200 hours ($\eta_{\mathrm{T=230^\circ C}}$ = 200 hours). For T_N = 80°C, the 0.01%-percentile, $t_N^{0.01\%}$, is given in the sixth column of Table 4.5 as a function of the underlying distribution function, the activation energy E_a and the dispersion parameter σ.

In the previous sections, it was found that the estimates of the model parameters η and σ deviate from their original values due to temperature gradients. It was found that the estimates of the parameters η and σ increased with increasing $\mathrm{T_{grad}}$.

From equation (4.7), it can be observed that when increasing η, t_N^p will increase as well. When increasing σ, t_N^p will decrease, since $\mathrm{F_N^{-1}}(p)$ is assumed to be negative.

Model Parameters			Highest significant slopes		$t_N^{0.01\%}$		
Distr.	E_a (eV)	σ	$b_\eta^{\max}$ (hours/°C)	$b_\sigma^{\max}$ or $b_\beta^{\min}$ (/°C)	True value (days)	Max. decr. (%)	Max. incr. (%)
Logn.	0.7	0.5	0.26	$16*10^{-4}$	$12*10^{2}$	-2.9	+0.65
Logn.	0.7	1	0.43	$23*10^{-4}$	$19*10^{1}$	-4.1	+1.1
Logn.	0.7	1.5	0.28	$24*10^{-4}$	$30*10^{0}$	-4.4	+0.70
Logn.	1.5	0.5	0.48	$35*10^{-4}$	$32*10^{5}$	-6.3	+1.2
Logn.	1.5	1	0.51	$29*10^{-4}$	$49*10^{4}$	-5.2	+1.3
Logn.	1.5	1.5	0.55	$40*10^{-4}$	$77*10^{3}$	-7.2	+1.4
Weib.	0.7	2	0.26	$-37*10^{-5}$	$80*10^{-6}$	-6.6	+0.65
Weib.	0.7	0.5	0.18	$-36*10^{-4}$	$80*10^{0}$	-4.1	+1.0
Weib.	0.7	0.2	0.32	$-35*10^{-3}$	$13*10^{2}$	-6.5	+1.9
Weib.	1.5	2	0.48	$-57*10^{-5}$	$20*10^{-2}$	-10	+1.2
Weib.	1.5	0.5	0.43	$-11*10^{-3}$	$20*10^{4}$	-12	+3.4
Weib.	1.5	0.2	0.50	$-11*10^{-2}$	$32*10^{5}$	-20	+5.1

Table 4.5: Highest significant b's among all combinations of q, N_j, stress profile and degree of censoring, as a function of the underlying distribution, the activation energy E_a and the dispersion parameter σ. The worst-case increase and the worst-case decrease of the estimated $t_N^{0.01\%}$ for T_{grad} = 5°C is given as well.

In the last two columns of Table 4.5, the maximal shift of $t_N^{0.01\%}$ in both directions is given for T_{grad} = 5°C. These shifts can be obtained from equation (4.7):

$$\text{Max. increase}(\%) = \frac{t_N^p(\eta_T + b_\eta^{\max} * T_{grad}, E_a, \sigma) - t_N^p(\eta_T, E_a, \sigma)}{t_N^p(\eta_T, E_a, \sigma)} * 100\% \qquad (4.8)$$

$$\text{Max.decrease}(\%) = \frac{t_N^p(\eta_\mathrm{T}, E_a, \sigma + \Delta\sigma) - t_N^p(\eta_\mathrm{T}, E_a, \sigma)}{t_N^p(\eta_\mathrm{T}, E_a, \sigma)} * 100\% \tag{4.9}$$

where $\sigma + \Delta\sigma = \sigma + b_\sigma^{\max} * \mathrm{T_{grad}}$ for experiments with an underlying lognormal distribution and $\sigma + \Delta\sigma = 1/\left(\beta + b_\beta^{\min} * \mathrm{T_{grad}}\right)$ for experiments with an underlying Weibull distribution.From Table 4.5, it can be observed that for experiments with lognormally distributed failure times, the worst-case decrease of $t_N^{0.01\%}$ amounts to -7.2% of $t_N^{0.01\%}$, while the worst-case increase of $t_N^{0.01\%}$ amounts to +1.4% of $t_N^{0.01\%}$. When choosing $\mathrm{T_{grad}}$ = 3°C, these values become –4.4% and +0.83%, respectively. For $\mathrm{T_{grad}}$ = 1°C, one has –1.5% and +0.3%, respectively.

For experiments with Weibull distributed failure times, it can be found that, for $\mathrm{T_{grad}}$ = 5°C, the worst-case decrease of $t_N^{0.01\%}$ amounts to -20% of $t_N^{0.01\%}$, while its worst-case increase amounts to +5.1% of $t_N^{0.01\%}$. When choosing $\mathrm{T_{grad}}$ = 3°C, these values become -12% and +3.0%, respectively. For $\mathrm{T_{grad}}$ = 1°C, one has –4.1% and +1.0%, respectively.

From these values, it can be concluded that the problem of temperature gradients is worse for experiments with Weibull than for experiments with lognormally distributed failure times. It is important to mention that the fact that low percentiles can be overestimated is worse than the fact that they can be underestimated. An underestimation just leads to a conservative estimate, while an overestimate can lead to too optimistic conclusions.

So, the most important conclusion from the simulations presented here is the fact that stress gradients can lead to an overestimate of the predicted lifetimes. Overestimates of low percentiles as high as 5.1% with respect to their original value have been observed. This value is in most practical cases negligible. Nevertheless, in order to get better estimates of the highest significant slopes b_η and b_β or $b_\sigma$and for finding explicit relations between these parameters and the parameters of Table 4.4, more simulations will have to be performed at more levels for these parameters.

4.4. Conclusions

In this chapter, a closer look has been taken at the assumptions made by the technique proposed in chapters 2 and 3.

First, an objective, purely statistical method has been developed for making the distinction between the lognormal and the Weibull distribution. This technique was based on the ratio between the Pearson correlation coefficient of the Weibull and the lognormal probability plot of the data set under consideration. It was found that this ratio was a pivotal quantity. Methods for testing H_0: Weibull versus H_A: Lognormal and H_0: Lognormal versus H_A: Weibull are proposed. These methods were compared with those described in the literature and it was found that the power of the new test was highest for most practical cases.

Second, the influence of stress gradients on the outcome of reliability experiments has been investigated. Restriction was made to temperature storage experiments. Several parameters defining a reliability experiment were considered. It was found that the influence of stress gradients in terms of percentage on the predicted lifetime was rather limited.

5. Conclusions

This work dealt with a problem that can best be defined as: "How to plan reliability experiments such that the total number of available test components and the total amount of available measurement time can be optimally used for predicting the reliability of the test components under consideration".

More precisely, a method has been developed for planning type I singly censored reliability experiments with two stress factors. The method was based on the expected asymptotic variance (EAV) of several important reliability parameters. These EAV's could be calculated using the expected total Fisher information matrix $\mathcal{I}$. Making use of the EAV's, a so-called optimum plan function (OPF) was constructed. Using this OPF, optimum plans could be calculated using numerical optimization techniques.

Two major conclusions concerning the obtained plans could be drawn. First, it was found that it is best to perform experiments at only 3 stress conditions and to perform measurements at only two different levels for each stress factor. This was in agreement with the literature dealing with the one stress factor problem. Another conclusion of these papers was that it is better to measure more DUT's at low stress levels than at high ones. In this work, it was found that this conclusion does not hold for the two stress factor problem anymore. This has been explained by the fact that our method does not assume equal measurement times per stress level.

The influence of increasing the total measurement time, the total number of available DUT's and the highest possible stress levels on the uncertainty of the predicted lifetime was investigated. It turned out that all these four suggestions significantly reduce this uncertainty.

The first shortcoming of the new technique was that the obtained plans were not able to detect departures from the assumed stress-life relationship and that they were not robust to departures from the assumed θ_0, θ_1, θ_2 and σ. In order to account for this disadvantage, suggestions were made for forcing the plans to have 4 or 5 stress conditions. For one of the two stress factors, these extra stress conditions were forced between the lowest and the highest stress condition. For the other stress factor, the extra conditions were forced to take the highest possible value. New optimal plans corresponding to these restrictions have been calculated and these plans were compared with the plans having three stress conditions, the so-called $q3$-plans. Is was observed that the new plans did not differ substantially from the $q3$-plans with respect to the choice of the stress conditions, the choice of the measurement time per stress condition and the choice of the proportion of the total amount of DUT's assigned to each stress condition. It has also been observed that the width of the approximate expected confidence intervals of the parameters η_H, n, E_a, σ and t_N^p did not become considerably wider.

The new plans turned out to be less sensitive to misspecifications of the model parameters θ_1 and θ_2, but with respect to the model parameter θ_0, the $q3$-plan turned out to be more robust. Concerning the robustness to misspecification of the dispersion parameter σ, no difference was found between the $q3$-plans and the new plans.

A the end of this work, a closer look has been taken at the assumptions made by the new method.

First, an objective, purely statistical method has been developed for making the distinction between the lognormal and the Weibull distribution. This technique was based on the ratio between the Pearson correlation coefficient of the Weibull and the lognormal probability plot of the data set under consideration. It was found that this ratio was a pivotal quantity. Methods for testing H_0: Weibull versus H_A: Lognormal and H_0: Lognormal versus H_A: Weibull were proposed. These methods were compared with those described in the literature and it was found that the power of the new test was highest.

Second, the influence of stress gradients on the outcome of reliability experiments has been investigated. Restriction was made to temperature storage experiments. Several parameters defining a reliability experiment were considered. It was found that the influence of stress gradients in terms of percentage on the predicted lifetime was rather small.

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14. Dreesen R., Croes K., Manca J., De Ceuninck W., De Schepper L., Pergoot A. and Groeseneken G., "Modeling hot-carrier degradation of LDD NMOSFETs by using a high-resolution measurement technique", accepted for presentation at ESREF'99

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Appendix A: Terminology and abbreviations

Activation energy	The energy necessary for an ion to jump from one lattice position to another. Also defined as the energy for diffusion that accounts for both the energy requirement and the neighboring vacancy requirement [MAL97] (unit: electron Volts or eV).
Al	Aluminum
Censoring time	The event time corresponding to a removal.
Continuous monitoring	A reliability experiment is said to be continuously monitored if both the failure times of the failed DUT's and the censoring times of the removed DUT's are exactly known.
Cu	Copper
DUT	Device Under Test.
EACV	Expected Asymptotic Covariance
EAV	Expected Asymptotic Variance of
ESD	Electrostatic Discharge.
ENF	Expected Number of Failures.
Failure time	Time at which a DUT meets a predefined FC. The event time corresponding to a failure.
FC	Failure Criterion.
FIT-rate	The FIT-rate of a distribution of failure times is defined as the hazard rate, but now in terms of 10^9 device-hours. So, 1 FIT = 1 failure/10^9 device-hours.
IC	Integrated Circuit. Also called chip.
Interval censoring	Interval monitored DUT's are inspected for failure only after every predefined period of time.
Kolmogorov-Smirnov test	Test used for testing the equality of two distributions.
MLE	Maximum Likelihood Estimation/Estimator/Estimate.
MND	Minimum Number of Devices.
μm	Micrometer or micron (1μm=10^{-6}m).
MMNF	Minimum Mean Number of Failures.
MOSFET	Metal Oxide Semiconductor Field Effect Transistor.

MPT	Minimal Percentage Time: Minimal percentage of the total test time that should be measured per stress condition.
MTTF test	Median Time To Failure test.
OPF	Optimum Plan Function.
Pivotal quantity	A pivotal quantity is a random variable $t(X_1, ..., X_n ; \theta)$, which is a function of $X_1, ..., X_n$ and whose distribution does not depend on θ.
p-value	The p-value of the outcome of a test H_0 versus H_A is the probability that this outcome could have been more extreme that the observed one when H_0 is true. Large p-values support H_0 while small p-values support H_A.
Si	Silicon
SiO_2	Silicon dioxide
TDDB	Time Dependent Dielectric Breakdown.
Triple point	Position on an on-chip interconnect where grain boundaries meet.
TT	Total test time = $\sum_{j=1}^{q} T_j$.
Type I singly censored data set	A data set obtained after performing a type I singly censored experiment. Note that for such data set there can be some DUT's censored before the last failure occurred.
Type I singly censored experiment	Type of reliability experiment. The lifetime measurement of the DUT's stressed at the same stress condition starts at the same time. The measurement is stopped after a predefined total measurement time.
Type II singly censored experiment	Type of reliability experiment. The lifetime measurement of the DUT's stressed at the same stress condition starts at the same time. The measurement is stopped when a predefined number of DUT's have failed.
W	Tungsten
x%-percentile	The x%-percentile of a distribution of failure times is the time at which x% of the population will have failed.

Appendix B: Notation list

α	Significance level of a statistical test.
$\bar{a}$	Vector (a_0 a_1 a_2 a_3) used for calculating the EAV of reliability parameters.
A(u), B(u), C(u)	Functions used for defining the expected total Fisher information matrix $\mathcal{I}$
b	Fitting parameter of the Takeda model and of the model of Hu.
$b_\eta, b_{E_a}, b_\beta, b_\sigma$	Slopes of the lines that model the linear influence of T_{grad} on the median of the estimated η's, E_a's, β's and σ's, respectively.
$\hat{b}_\eta, \hat{b}_{E_a}, \hat{b}_\beta, \hat{b}_\sigma$	Estimates of the parameters b_η, b_{E_a}, b_β and b_σ, respectively.
β	Shape parameter of the Weibull or the lognormal distribution function.
C	Fitting parameter of different acceleration models. Used in the model of Hu, in the model of Takeda, in the 1/E-model, in the E-model, in Black's model and in Arrhenius' equation (unit: same as η).
cE	cE is defined as the number of censored components in case the experiment is stopped at the time all components have failed. In mathematical notation: $cE = N\text{-}r(\text{time} = \infty)$.
cEb	Number of samples that were removed before the r-th failure.
d	Average grain diameter.
δ	A 0-1 variable which is defined to be 1 if a DUT has failed and 0 if it has been removed.
$DE_W(\rho_W/\rho_L)$	The estimated density curve of ρ_W/ρ_L for Weibull samples.
$DE_L(\rho_W/\rho_L)$	The estimated density curve of ρ_W/ρ_L for lognormal samples.
E	Electric field (unit: MV/cm).
E_a	Activation energy. Fitting parameter used in Black's model and in the Arrhenius' equation (unit: eV).
f_X	Density function of the random variable X.
F_X	Distribution function of the random variable X.
γ	Fitting parameter of the E-model.
G	Fitting parameter of the 1/E-model.
H	Subscript H refers to the highest possible stress level.

H_0	Null hypothesis of a statistical test.
H_A	Alternative hypothesis of a statistical test.
$\mathcal{I}$	Expected total Fisher information matrix of a type I singly censored reliability experiment
i,j,k	Used for indexing.
I_d	Drain current
$I_{d,lin}$, V_t, $I_{d,sat}$, $G_{m,max}$ and I_{cp}	Possible parameters to be measured for the determination of a MOSFET's lifetime.
I_{SUB}	Substrate current.
J	Current density (unit: MA/cm^2).
K	Constant used in the theory of Blech.
k_B	Boltzmann constant ($8.6*10^{-5}$eV/K).
l	Number of stress factors in a reliability experiment.
$\pounds$	Likelihood function.
L_{crit}	Critical values of the test H_0: Lognormal.
L_{Π}	Power of the test H_0: Lognormal.
λ_X	Hazard rate of the random variable X.
l_{max}	Length of the longest polycrystalline part in a semi-bamboo interconnect.
m	Length of a vector.
μ	Scale parameter of the extreme value or the normal distribution function.
η	Scale parameter of the Weibull or the lognormal distribution function. Also referred to as median life.
n	Fitting parameter of Black's model.
N_j	Total number of DUT's stressed at the j-th stress condition.
N	Total number of DUT's.
$_N$	Subscript N refers to the normal operation conditions.
p	Typical value to characterize a percentile-level.
Π	Power of a statistical test.
$\overline{P}$	Vector containing the re-scaled version of all unfixed parameters that define a type I singly censored experiment
P_j	Probability for a DUT stressed at the j-th stress condition to have failed at the end of the experiment.
$\overline{P}_{\overline{\theta}}$	Optimum plan when the model parameters are set to $\overline{\theta}$.

Π_j	Proportion of the total number of DUT's measured at the j-th stress condition.
q	Number of stress levels in a reliability experiment.
Θ_i	i-th fitting parameter of a lifetime model written in terms of time.
θ_i	i-th fitting parameter of a re-scaled lifetime model written in terms of log time.
$\bar{\theta}$	$(\theta_0\ \theta_1\ \theta_2\ \sigma)$
ρ	Pearson's correlation coefficient of the points on a plot.
r_j	Total number of failed DUT's at the j-th stress condition.
ρ_W	The Pearson correlation coefficient of the points on the Weibull plot of the experiment under consideration.
ρ_L	The Pearson correlation coefficient of the points on the lognormal plot of the experiment under consideration.
σ	Dispersion parameter.
T	Absolute temperature (unit: degrees K).
τ	Random variable.
T_{grad}	The maximal temperature difference between two oven positions.
$t^e_{k,j}$	Event time of the k-th sample stressed at the j-th stress condition.
T_j	Total measurement time at the j-th stress condition.
t^p_N	The p%-percentile of the distribution function of failure times under normal operation conditions.
t_{ox}	Oxide tickness.
T_{target}	Target temperature
V_d	Drain Voltage.
V_g	Gate Voltage.
w	Width of an on-chip interconnection. Also called line width.
W	Weight vector for the OPF function.
W_{crit}	Critical values of the test H_0: Weibull.
W_Π	Power of the test H_0: Weibull.
$\mathbf{x}$	Numerical vector.
X	Random variable.
$\Xi_{i,j}$	Value of the i-th stress factor at the j-th stress condition when the lifetime model is written in terms of time.

$\xi_{i,j}$	Value of the i-th stress factor at the j-th stress condition when the lifetime model is written in terms of log time and is re-scaled. Value between 0 and 1.
$\mathbf{y}$	Numerical vector.
Y	Random variable.
Y_j	Natural logarithm of the total measurement time at the j-th stress condition.
$y^e_{k,j}$	Logarithm of event time $t^e_{k,j}$.
y^p_N	The log p%-percentile of the distribution function of failure times under normal operation conditions.
z_j	Standardized total measurement time at the j-th stress condition.

APPENDIX C: The number of random $\overline{P}$-vectors used for the determination of MEAS(i) in the OPF

In chapter 2 of the main text, the OPF function is defined as follows:

$$\mathrm{OPF}(\overline{P}) = \sum_{i=1}^{6} W(i) \frac{\mathrm{TERM}(i)}{\mathrm{MEAS}(i)}$$

The six terms were defined in section 2.3.3.1 and a method for finding a measure MEAS(i) for each term TERM(i) was proposed in section 2.3.3.2. This measure was obtained as follows:

1. Simulate NRP random $\overline{P}$-vectors (NRP ≡ Number of Random vectors $\overline{P}$).
2. For each $\overline{P}$-vector, calculate TERM(i) for $i = 1, \ldots, 6$.
3. Set MEAS(i) to the median of each TERM(i) for $i = 1, \ldots, 6$.

In the main text, NRP is set to 100. This choice will be motivated in this appendix.

A major requirement of this measure is that it does not significantly depend on the simulated random vectors $\overline{P}$. Of course, the higher NRP, the more stable the measure will be. Figures C.1 and C.2 contain the standard deviations of the measures MEAS(i), $i = 1, \ldots, 6$ as a function NRP. For each value of NRP, the measures MEAS(i), $i = 1, \ldots, 6$ were obtained 100 times. The standard deviations of these 100 MEAS(i)'s were then plotted as a function NRP in figures C.1 and C.2. The difference between the two figures is the choice of the parameters contained in parameter set 1-3 of table 2.2 in chapter 2 of the main text. The values for these parameters are summarized in table C.1 for both figures. For illustrative purposes, all parameter values are given for the specific case of Black's equation. In table C.1, the values for MEAS(i) calculated using NRP=10,000 are also given. This is done just to give an idea of the order of magnitude of each MEAS(i), $i = 1, \ldots, 6$.

Figure C.1	Figure C.2
$T_H = 200°C$ and $J_H = 8$ MA/cm²	$T_H = 250°C$ and $J_H = 5.5$ MA/cm²
$T_N = 125°C$ and $J_N = 0.2$ MA/cm²	$T_N = 80°C$ and $J_N = 0.2$ MA/cm²
$\eta_H = 50$ hours	$\eta_H = 20$ hours
$n = 2$	$n = 1$
$E_a = 1.3$ eV	$E_a = 0.7$ eV
$\sigma = 0.7$	$\sigma = 0.5$
Underlying WEIBULL distribution	Underlying LOGNORMAL distribution
TT = 500 hours ; $N = 100$	TT = 1000 hours ; N = 300
$p = 0.01\%$	$p = 0.1\%$
MND = 5 ; MMNF = 3 ; MPT = 1%	MND = 5 ; MMNF = 3 ; MPT = 1%
$q = 3$	$q = 3$
MEAS(1) = 0.024	MEAS(1) = 0.0031
MEAS(2) = 15.19	MEAS(2) = 0.19
MEAS(3) = 9.58	MEAS(3) = 0.89
MEAS(4) = 0.0080	MEAS(4) = 0.00065
MEAS(5) = 11.66	MEAS(5) = 0.58
MEAS(6) = 36.94	MEAS(6) = 1.51

Table C.1: Values of the parameters of parameter sets 1-3 described in table 2.2 of the main text used for constructing figure C.1 and C.2. The values for MEAS(i) calculated using NRP=10,000 are also given.

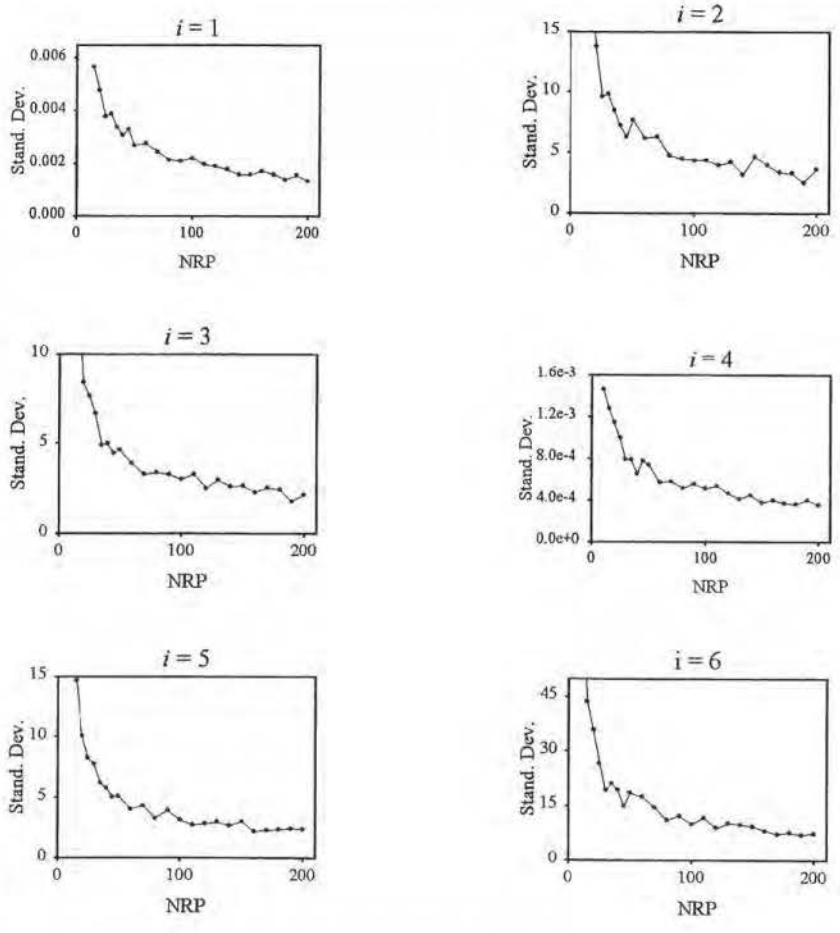

Figure C.1: Standard deviation of MEAS(i) *for i = 1, ..., 6 as a function of NRP. The values of the parameter of sets 1-3 described in table 2.2 of the main text have been set to those given in the first column of table C.1.*

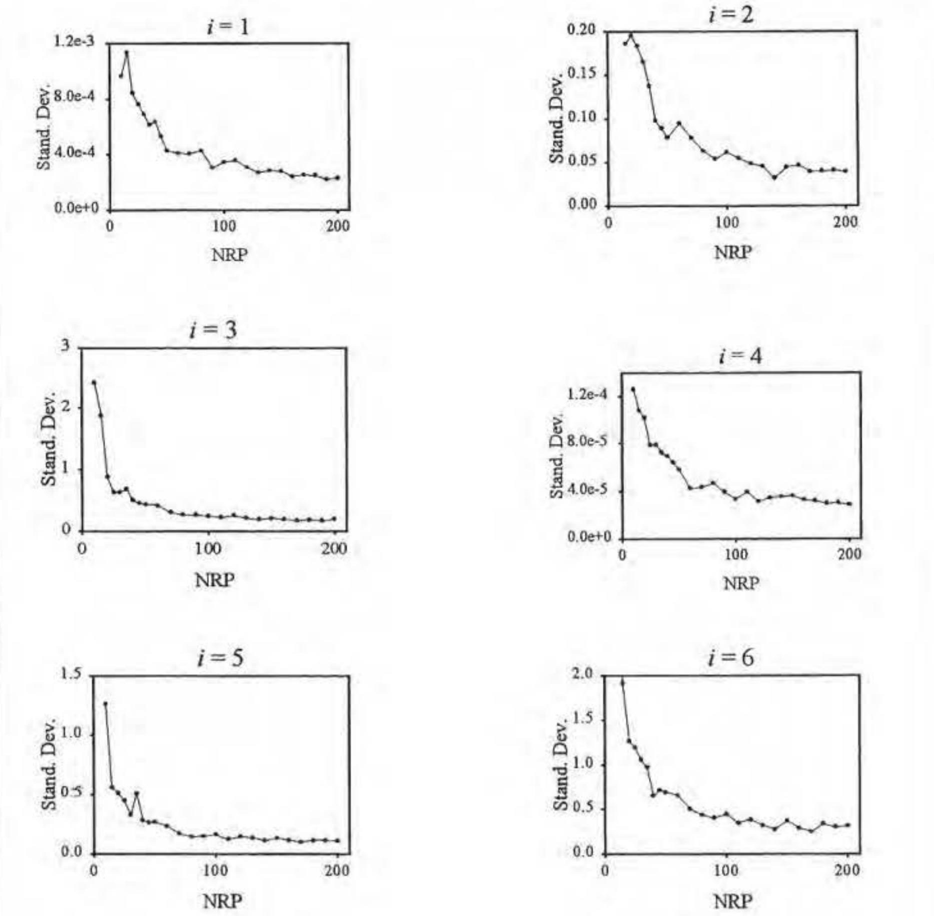

Figure C.2: Standard deviation of MEAS(i) *for i = 1, ..., 6 as a function of NRP. The values of the parameter of sets 1-3 described in table 2.2 of the main text have been set to those given in the second column of table C.1.*

APPENDIX D: Approximate expected confidence intervals on the parameters η_H, n, E_a, σ and t_N^p

An approximate expected asymptotic confidence interval of θ_0 is given by $\theta_0 \pm 2 * \sqrt{\mathrm{EAV}(\theta_0)}$. Since $\theta_0 = \ln(\eta_H)$, an approximate 95% confidence interval of η_H can be obtained using

$$[\eta_H * \exp\left(-2 * \sqrt{\mathrm{EAV}(\theta_0)}\right), \eta_H * \exp\left(+2 * \sqrt{\mathrm{EAV}(\theta_0)}\right)]$$

Using basic statistical theory and the calculations given in section 2.2.3 of the main text, it can be calculated that the EAV of the factor n can be obtained using

$$\mathrm{EAV}(n) = \frac{\mathrm{EAV}(\theta_1)}{\left[\ln\left(\mathrm{j}_N / \mathrm{j}_H\right)\right]^2}$$

An approximate 95% confidence interval of n is given by $n \pm 2 * \sqrt{\mathrm{EAV}(n)}$. The EAV of the activation energy E_a can be calculated using

$$\mathrm{EAV}(E_a) = \frac{\mathrm{EAV}(\theta_2)}{\left[\frac{1}{\mathrm{k_B}}\left(\frac{1}{\mathrm{T}_N} - \frac{1}{\mathrm{T}_H}\right)\right]^2},$$

allowing the calculation of the following approximate 95% confidence interval: $E_a \pm 2 * \sqrt{\mathrm{EAV}(E_a)}$.

For the parameter σ, an approximate 95% confidence interval is obviously given by $\sigma \pm 2 * \sqrt{\mathrm{EAV}(\sigma)}$.

The log p%-percentile of the distribution function of failure times under normal operation levels, $y_N^p = \ln(t_N^p)$, can be obtained using equation (2.16). An approximate 95% confidence interval of t_N^p can be obtained using

$$[t_N^p * \exp\left(-2 * \sqrt{\mathrm{EAV}\left(y_N^p\right)}\right), t_N^p * \exp\left(+2 * \sqrt{\mathrm{EAV}\left(y_N^p\right)}\right)]$$

Samenvatting

Inleiding en doelstellingen

Dit werk kan best gesitueerd worden in het onderzoek naar het testen van de betrouwbaarheid van geïntegreerde schakelingen (IC's). Deze testen zijn belangrijk omdat enerzijds vanuit het oogpunt van veiligheid continu operationele IC's onontbeerlijk zijn, en anderzijds omdat de concurrentiekracht van fabrikanten sterk afhangt van de aantoonbare betrouwbaarheid van de geproduceerde IC's. De toenemende miniaturisatie in de microtechnologie zorgt ervoor de betrouwbaarheid van een IC steeds moeilijker te garanderen is.

De betrouwbaarheid van een IC wordt meestal gecontroleerd door na te gaan of alle verschillende soorten componenten op de IC betrouwbaar zijn. Deze componenten kunnen opgesplitst worden in off-chip en on-chip onderdelen. Op het on-chip niveau moeten zowel de actieve als de passieve componenten getest worden. Enkele van die on-chip elementen zijn MOSFET's [VUI98], diëlektrica [MAR98], interconnecties [MAL97], ...

Een cruciaal probleem dat opduikt bij het testen van een nieuwe technologie is dat de levensduur van de geteste componenten onder normale werkingscondities zeer lang is (in orde van jaren). Daarom is het nodig dat de fysische mechanismen die verantwoordelijk zijn voor faling grondig bestudeerd worden en moeten er methodes ontwikkeld worden die deze mechanismen versnellen. Ook moeten er modellen ontwikkeld worden om de resultaten die bekomen zijn op de versnelde condities te extrapoleren naar normale werkingscondities.

Bij het testen van een nieuwe technologie worden meestal de volgende stappen gevolgd:

- Ontwerp een teststructuur die toelaat de levensduur van de nieuwe component te meten. Die levensduur wordt meestal gedefinieerd aan de hand van een bepaalde elektrische karakteristiek van de component. Een interconnectie wordt bijvoorbeeld als gefaald beschouwd vanaf het moment dat diens weerstandstijging een vooraf gedefinieerde waarde overschrijdt.
- Kies één of meerdere stressfactoren die het verouderingsgedrag van de nieuwe component versnelt. Zo een stressfactor wordt vooral bepaald door het type

component dat bestudeerd wordt. Typische stressfactoren zijn: verhoogde of verlaagde temperatuur, verhoogde stroomdichtheid, verhoogde spanning, ...

- Meet de faaltijd van een aantal componenten bij enkele verhoogde stressniveaus.
- Bepaal het model voor het extrapoleren van deze faaltijden naar normale werkingscondities. Zo een model bestaat uit een verdelingsfunctie van de faaltijden enerzijds en een relatie tussen deze faaltijden en de aangelegde stress anderzijds. De keuze van zulke modellen is meestal gebaseerd op ervaringen met vorige technologieën.
- Aanvaard of verwerp de nieuwe technologie op basis van de statistische analyse van de resultaten.

In deze tekst werden betrouwbaarheidsexperimenten op on-chip interconnecties gebruikt om de nieuw ontworpen technieken te illustreren. De twee meeste gebruikte stressfactoren bij het testen van zulke interconnecties zijn een verhoogde temperatuur T en een verhoogde stroomdichtheid J. Als levensduurmodel is het model van Black ongetwijfeld het meest gebruikte. Dit model geeft de levensduur η als functie van T en J:

$$\eta(\mathrm{T},\mathrm{J}) = C * \mathrm{J}^{-n} * \exp(\frac{E_a}{\mathrm{k_B T}})$$

Waar C, n en E_a onbekende parameters zijn. C is een materiaal parameter (eenheid: zelfde als η) en E_a is gedefinieerd als de activeringsenergie (eenheid: eV). $\mathrm{k_B}$ is de constante van Boltzmann.

Het doel van een betrouwbaarheidsexperiment is het schatten van de modelparameters. Voor het Black-model zijn dat de parameters C, n en E_a. Ook de spreidingsparameter σ van de onderliggende verdelingsfunctie van faaltijden is zo een modelparameter. De twee verdelingsfuncties die in dit werk werden beschouwd zijn de Weibull en de lognormale verdeling. Zij werden in dit werk dan ook uitgebreid besproken.

Het schatten van deze modelparameters gebeurt in dit werk met behulp van de zogenaamde Maximum Likelihood Estimation (MLE) techniek. Ook deze techniek kwam in dit werk ruim aan bod. Belangrijk hier was dat de schattingen gebaseerd waren op de zogenaamde likelihood functie. Deze functie geeft "de kans op het experiment" als functie van de te schatten parameters.

Het voorliggend werk was actief op het domein van het plannen van betrouwbaarheidsexperimenten. Het plannen van een betrouwbaarheidsexperiment heeft als doel om de stressniveaus, het aantal gemeten componenten per stressniveau en de totale meettijd per stressniveau zodanig te kiezen dat de betrouwbaarheid van de geteste componenten kan bepaald worden binnen een vooropgestelde nauwkeurigheid.

In dit doctoraat hebben we ons beperkt tot het plannen van betrouwbaarheidsexperimenten met twee stressfactoren. Dit omdat het één-stress-factor-probleem al veelvuldig bestudeerd werd en omdat experimenten met meer dan twee stressfactoren zelden of nooit worden uitgevoerd.

De nieuwe techniek

Het hoofddoel van dit doctoraat is het plannen van zogenaamde type I singly censored experimenten met twee stressfactoren. In een type I singly censored experiment worden de metingen van alle samples die gemeten worden op eenzelfde stressniveau tegelijkertijd gestart (singly censored). De metingen worden beëindigd na een vooraf ingestelde tijd (type I).

Een voorname eigenschap van de MLE-techniek is dat ze onder andere toelaat om de onzekerheid op bepaalde modelparameters op voorhand te schatten, tenminste als het aangenomen levensduurmodel lineariseerbaar is. Dit is echter het geval voor de meeste levensduurmodellen. Als maat voor de onzekerheid op de schatting van een modelparameter werd gebruik gemaakt van de zogenaamde Expected Asymptotic Variance (EAV).

De EAV kon berekend worden aan de hand van de Fisher informatie matrix $\mathcal{I}$. Deze $\mathcal{I}$ was gebaseerd op de tweede afgeleide van de likelihood functie.

Alle parameters die rechtstreeks met het experimenteel plan te maken hadden, werden getransformeerd naar variabelen tussen 0 en 1 en werden gegroepeerd in de zogenaamde plan-vector $\overline{P}$. Deze parameters zijn de volgende:

- De stressniveaus.
- De meettijd per stressniveau.
- Het aantal gemeten samples per stressniveau.

De Optimale Plan Functie (OPF) werd opgesteld. Deze OPF is een functie van de plan-vector $\overline{P}$. De OPF werd zó opgesteld dat het plan dat overeenkomt met het minimum van de OPF, als optimaal kan beschouwd worden.

De OPF werd opgesteld als een som van zes termen:

$$\mathrm{OPF}(\overline{P}) = \sum_{i=1}^{6} W(i) \frac{\mathrm{TERM}(i)}{\mathrm{MEAS}(i)}$$

Iedere term, TERM(i), representeerde de onzekerheid op een bepaalde parameter die van invloed was op de betrouwbaarheid van de geteste componenten. Elke MEAS(i) was een maat voor TERM(i), dit om ervoor te zorgen dat de verhouding TERM(i)/MEAS(i) van eenzelfde grootteorde was voor iedere i. $W(i)$ werd ingevoerd om de gebruiker de kans te geven om bepaalde termen meer gewicht te geven dan een andere teneinde een plan te vinden dat voldoet aan de specifieke vereisten van het experiment.

Het minimum van de OPF diende numeriek bepaald te worden. Daarom werd de OPF geprogrammeerd in GAUSS.

De twee voornaamste conclusies die konden getrokken worden met betrekking tot de hier geplande experimenten zijn de volgende:

- Het is best om experimenten uit te voeren op slechts drie verschillende stressniveaus. Voor iedere stressfactor is het best om tweemaal te meten op het hoogste mogelijke niveau en één maal op een lager niveau. Bij het stressniveau waarop de ene stressfactor op het laagste niveau staat, moet het stressniveau van de andere stressfactor op zijn hoogste niveau gezet worden. Het feit dat het best is te meten bij slechts twee verschillende niveaus per stressfactor werd reeds opgemerkt in de literatuur die het één-stress-factor-probleem beschouwd.
- We konden ook besluiten dat meer samples moeten gemeten worden op het hoogst mogelijke stressniveau. Dit was niet in overeenstemming met de literatuur die het één-stress-factor-probleem beschouwd. Dit feit werd verklaard door het feit dat in die papers een gelijke meettijd per stressniveau werd beschouwd.

De invloed van het verhogen van de totale meettijd, van het totaal aantal samples en van de hoogste mogelijke stressniveaus op de onzekerheid van het geplande levensduur werd bestudeerd. We konden besluiten dat al deze vier suggesties een grote invloed hadden op de voorspelde onzekerheid.

Een kritische kijk op de nieuwe techniek

De twee voornaamste bezwaren op de geplande experimenten in het vorige deel, werden in dit deel nader bekeken en er werd gezocht naar een oplossing. De voornaamste bezwaren waren dat:

- De geplande experimenten niet toelieten om het aangenomen levensduurmodel te verifiëren
- De geplande experimenten niet robuust waren tegen misspecificaties van de modelparameters.

De oorzaak van deze tekortkomingen lag in het feit dat de geplande experimenten slechts drie verschillende stressniveaus hadden en slechts twee stressniveaus per stressfactor. Daarom werden in dit deel plannen voorgesteld die gedwongen werden om vier of vijf verschillende stressniveaus te hebben. De extra stressniveaus hadden de volgende eigenschap: voor een van de twee stressfactoren werd dit niveau gelegd tussen het hoogst mogelijke en het lage niveau uit het plan met drie stressniveaus (het zogenaamde $q3$-plan), terwijl voor de andere stressfactor het hoogste mogelijke stressniveau werd gekozen. De nieuwe optimale plannen die gevolg gaven aan deze beperkingen werden vergeleken met het $q3$-plan. We konden zien dat de nieuwe plannen niet zo erg verschillenden van het $q3$-plan met betrekking tot de stressniveaus, de meettijd per stressniveau en het aantal gemeten samples per stressniveau. Ook de verwachte onzekerheid op de modelparameters steeg niet gevoelig.

De nieuwe plannen werden dan geëvalueerd op het gebied van gevoeligheid aan misspecificaties van de modelparameters. We vonden dat de nieuwe plannen gevoeliger waren aan het misspecifiëren van de modelparameter η_H, maar dat ze minder gevoelig waren aan de modelparameters E_a en n. Het dient opgemerkt dat vooral dit laatste feit belangrijk is omdat deze twee modelparameters werden gebruikt voor het extrapoleren naar normale werkingscondities.

Commentaren op de aannames die gemaakt werden in dit doctoraat

In dit doctoraat werden twee voorname aannames gemaakt:

- Er werd verondersteld dat de verdelingsfunctie van faaltijden op voorhand gekend was.
- Er werd aangenomen dat de aangelegde stress gelijk was aan de geplande stress.

In dit doctoraat werd er een methode ontwikkeld die toelaat een keuze te maken tussen de lognormale en de Weibull verdeling. Deze techniek was gebaseerd op de verhouding tussen de Pearson correlatiecoëfficiënt van de Weibull en de lognormale waarschijnlijkheidsplot van de beschouwde data set. Het werd bewezen dat deze verhouding pivotaal was. Er werden methodes voorgesteld voor het testen van H_0: Weibull versus H_A: Lognormaal en H_0: Lognormaal versus H_A: Weibull. Deze methodes werden vergeleken met die uit de literatuur en we zagen dat de power van de nieuwe methode hoger was.

Ook werd de invloed van zogenaamde stressgradiënten op de schattingen van de modelparameters bestudeerd. We hebben ons hier beperkt tot de studie van zogenaamde temperature storage experimenten (met als enige stressfactor temperatuur). Het gedrag van verscheidene parameters die een invloed hebben op een betrouwbaarheidsexperiment werden bestudeerd. We konden besluiten dat de invloed van stressgradiënten een eerder geringe invloed had op de voorspelde levensduur.