The characteristics of photon and phonon standing waves in a periodic medium

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Abstract

The main characteristics for the spatial variation of the photon and phonon wave fields at the band gap boundaries are derived for a one dimensional medium with periodic optical or acoustic parameters. The derivations are based on symmetry considerations and on analytical results derived from the basic differential equation for the wave field. A simple relation is derived between the band gap width and the derivative of the square modulus of the field at the interface between the regions of high and low wave velocity. The features of the standing waves are derived for various gap numbers and continuously varying acoustic/optical parameters of the medium. Using the results a remarkable asymmetric behaviour of the wave absorption near the Brillouin-zone boundaries could be explained in a straightforward way.

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1. Introduction

In recent years a lot of research has been done on waves in media with periodic properties, both for scientific and technical reasons. This can concern acoustic waves in so called phononic crystals (e.g. acoustic superlattices) or electromagnetic waves in photonic crystals, where the optical parameters have translational symmetry. Because an electron moving in a periodic potential and an electromagnetic (or acoustic) wave in a periodic medium are related phenomena, it can be understood that the wave frequencies will show band structure, and that forbidden frequency intervals (frequency gaps) will occur [1-4]. This is the physical basis for a number of existing or possible applications such as antennas, lasers, optical filters, prisms, mirrors, wave guides etc. (See e.g. [5] and references given there). Near the band edges interesting phenomena occur, such as the suppression of the wave group velocity and the modification of the spontaneous emission rates of atoms and molecules [6-10].

In this paper special attention will be paid to the results of Kuzmiak et al. [11] where a periodic system was studied, consisting of thin metallic regions in combination with non dissipating regions. These authors found a remarkable behaviour of the absorption coefficients and electromagnetic wave lifetimes for wave vectors near the zone boundaries. These physical parameters showed asymmetric behaviour according to the frequency value (larger or smaller values depending on the position of the frequency with respect to the band gap, see § 5). In order to explain results like that of Kuzmiak et al. it is necessary not only to have a good knowledge of the photonic (phononic) band structure, but also to have a clear physical picture of the spatial variation of the (acoustic or electromagnetic) field for frequencies near the forbidden band. One can expect indeed that absorption effects will be the most important in spatial regions with a large field amplitude. The properties of periodic media can be studied in the most direct way in a one dimensional medium with alternate layers [12-21], in analogy with the Kronig - Penney treatment of an electron moving in a periodic potential [22,23]. The properties of the band frequency spectrum for such a one dimensional periodic medium were studied already by several authors [13,24]. In the present paper we concentrate rather on the properties of the field near the band edges (standing wave field), which are investigated systematically as a function of the parameters of the periodic structure. The explanation of the results of Kuzmiak et al. will be straightforward then.

In § 2 the results for the dispersion relation and the field will be given, using dimensionless quantities as much as possible. The band gap and field behaviour are studied in a more general analytic way in § 3 and 4, where also some numerical examples are given. Conclusions are drawn in § 5.

2. Dispersion relation and spatial variation of the field

In our study of the propagation of electromagnetic or elastic waves, we start with the one dimensional wave equation (Helmholtz eqn., see e.g. [1])

$$c^2(x)\,\frac{d^2U(x)}{dx^2} + \omega^2 U(x) = 0 \qquad (1)$$

The field U(x) represents the spatial dependence of e.g. an electric field component, or the deviation of an atomic coordinate from its equilibrium value. c(x) is the local periodic wave velocity which is chosen as a piecewise constant function, and ω the frequency. We adopt the following notation. In regions I (layers of thickness d_1) the velocity equals c_1 ; for regions II (thickness d_2) the velocity is c_2 with $c_2 < c_1$. Near the origin ($x = 0$) region I extends from $x = 0$ to d_1 and region II from $x = -d_2$ to 0 . The period for the function c(x) equals $d = d_1 + d_2$.

Due to the periodicity of c^2 in eqn. (1) the field U(x) is a Bloch type function :

$$U(x+d) = \exp(i\,k\,d)\,U(x) \qquad (2)$$

where k is the quasi-wave number. In principle k can be any number, but often it is chosen in the interval $[-\pi/d , \pi/d]$ (first Brillouin zone). In every region U(x) can be written as a linear combination of exponentials $\exp(\pm i\,k\,x)$. By taking into account 1) the continuity of U(x) and dU/dx at the interface points and 2) the Bloch condition, one finds a set of linear and homogeneous eqns. in the coefficients of the exponentials in each region. The zero determinant condition for this set of eqns. leads in a straightforward way to the dispersion relation

$$\cos k\,d = (\cos k_1 d_1)(\cos k_2 d_2) - \frac{1}{2}\left(\frac{k_1}{k_2} + \frac{k_2}{k_1}\right)(\sin k_1 d_1)(\sin k_2 d_2) \qquad (3)$$

where

$$k_j = \frac{\omega}{c_j} \qquad (j = 1, 2) \qquad (4)$$

are local wave numbers. For a given ω the function $U(x)$ can be determined from the homogeneous equations for the coefficients of the exponentials. In the regions I and II near the origin $U(x)$ is respectively proportional to :

$$U_I(x) \sim Q_1 \cos(\omega x / c_1) - (c_1 / c_2)\, Q_2 \sin(\omega x / c_1)$$

$$U_{II}(x) \sim Q_1 \cos(\omega x / c_2) - Q_2 \sin(\omega x / c_2) \qquad (5a,b)$$

where the constants Q_j will be given further.

In order to study the dependence of the dispersion relation and the field $U(x)$ on the various parameters (ω , c_1 , c_2 , d_1 , d_2) in a transparent way, dimensionless parameters will be used as much as possible. We introduce the following dimensionless parameters :

$$\alpha = c_1 / c_2 \qquad \beta = d_1 / d_2 \qquad (6a,b)$$

$$\omega^* = \frac{\omega\, d}{<c>} \qquad x^* = \frac{x}{d} \qquad k^* = k\, d \qquad (7a,b,c)$$

The quantity $< c >$ in eqn. (7a) is a weighted mean wave velocity in the regions I and II [25] defined by $< c >^{-1} = (d_1 / d) (c_1)^{-1} + (d_2 / d) (c_2)^{-1}$. Together with the introduction of dimensionless parameters, we transform the dispersion relation (3) in a more transparent equation. The product of the 2 cosines in this eqn. can be written as $\cos (k_1 d_1 + k_2 d_2)$ augmented with a correction, where the latter can be combined with the second contribution in eqn. (3). After these manipulations and the introduction of the dimensionless quantities of eqn. (6,7), one obtains for the dispersion relation

$$\cos k^* = \cos \omega^* - \frac{1}{2} \frac{(1-\alpha)^2}{\alpha} \left(\sin \frac{\beta\, \omega^*}{\alpha + \beta} \right) \left(\sin \frac{\alpha\, \omega^*}{\alpha + \beta} \right) \qquad (8)$$

The dimensionless parameters can as well be substituted in the expression for U (eqns. (5)). This leads to

$$U_I(x^*) \sim Q_1 \cos \frac{(1+\beta)\, \omega^*\, x^*}{\alpha + \beta} - \alpha\, Q_2 \sin \frac{(1+\beta)\, \omega^*\, x^*}{\alpha + \beta}$$

$$U_{II}(x^*) \sim Q_1 \cos \frac{\alpha\,(1+\beta)\, \omega^*\, x^*}{\alpha + \beta} - Q_2 \sin \frac{\alpha\,(1+\beta)\, \omega^*\, x^*}{\alpha + \beta} \qquad (9a,b)$$

where detailed study turns out that

$$Q_1(\omega^*) = -\cos k^* + ½ \left[(1-\alpha) \left(\cos \frac{(\beta-\alpha)\, \omega^*}{\beta+\alpha} \right) + (1 + \alpha) (\cos \omega^*) \right]$$

$$Q_2(\omega^*) = -\sin k^* - \frac{1}{2\alpha} \left[(1-\alpha) \left(\sin \frac{(\beta-\alpha)\, \omega^*}{\beta+\alpha} \right) + (1 + \alpha) (\sin \omega^*) \right] \qquad (10a,b)$$

For zero contrast between the media ($\alpha = 1$) dispersion relation (8) reduces to $\cos \omega^* = \cos k^*$. Hence in the extended zone scheme the dispersion relation becomes the straight line $\omega^* = k^*$. In the reduced zone scheme this corresponds to a number of straight lines intersecting at $k^* = 0$ or $\pm \pi$ (the centre and boundaries of the first Brillouin zone). For non zero contrast ($\alpha \neq 1$) band gaps for ω^* will open at the centre and boundaries of the Brillouin zone, because the term containing α in eqn. (8) brings the r.h.s. of the equation out of the interval [- 1 , + 1]. Each gap can be characterized by two boundary frequencies which we will denote as ω_{n-}^* and ω_{n+}^* . n is the number of the gap, and ω_{n-}^* and ω_{n+}^* are respectively the low and high ω^* value at the boundaries of the nth gap. The fields corresponding to these ω^* values will be denoted as U_{n-} and U_{n+} respectively. The successive gaps (n = 1 , 2 , ...) are attained for $k = n\pi$ and the frequency gaps are situated near $\omega^* \approx n\pi$.

3. Behaviour of ω^* and the band gaps

The band gap will disappear when the second term in the r.h.s. of eqn. (8) equals zero. This can occur in the trivial case $\alpha = 1$, but also if one of the sinus functions is zero. So the condition $\beta\,\omega^* / (\alpha + \beta) = n'\pi$ (integer n') leads to a zero gap situation. Because ω^* will then approximate $n\pi$ within a very narrow interval, one easily obtains the condition for zero gap

$$(\beta/\alpha)_{z.g.} = \frac{n'}{n - n'} \tag{11}$$

where n' is any integer for which $0 \leq n' \leq n$. For n' = 0 or n one has the limiting situation where $\beta/\alpha \rightarrow 0$ or ∞ (one material has an extreme thickness or c value). The gap disappears for β/α equal to 1 (for n = 2) , ½ and 2 (for n = 3) , 1/3 , 1 and 3 (for n = 4) etc. The n = 1 gap can not really disappear because there exists no appropriate n' value.

The values of $\omega_{n\pm}^*$ to be determined from dispersion eqn. (8) with the l.h.s. equal to $\pm$ 1, are functions of α and the ratio β/α . For a given α , the two real and positive numbers p and 1/p for the ratio β/α lead to the same values of $\omega_{n\pm}^*$. Hence the gap boundaries and the frequency gaps are functions of the quantity $|\ln \beta/\alpha|$.

In fig. 1 the behaviour of the gap width is illustrated for $n = 3$. The width $\Delta\omega_3^* = \omega_{3+}^* - \omega_{3-}^*$ is shown as a function of β/α for $\alpha = 1.5$. As mentioned above the gap disappears for $\beta/\alpha = 0, ½ , 2$ and ∞ . Because of the dependence on $| \ln \beta/\alpha |$, it shows a maximum for $\beta/\alpha = 1$, in between the zero gap situations. Other maxima appear for β/α near 0.25 and 4 . Some results on the behaviour of U_{n-} and U_{n+} will be discussed in relation to fig. 1.

4. Analysis of the behaviour of the fields U_{n-} and U_{n+}

4.1. Periodicity, number of oscillations and parity

Although $U(x)$ is known exactly by eqns. (9a,b) and (10a,b), it is difficult to infer its main features from it in a simple way. Therefore we will study various properties of $U_{n\pm}(x)$, which will lead finally to an overall picture of the behaviour of these functions.

Because $k^* = n\pi$ (modulo 2π) at the gap boundaries, the Bloch condition (2) and eqn. (7c) lead to

$$U_{n\pm}(x+d) = (-1)^n\, U_{n\pm}(x) \tag{12}$$

This implies that for gaps with even n , $U_{n\pm}$ has the same periodicity as $c(x)$, while for odd n it has a redoubled period. Both for n even or odd, the squared function (intensity) $U_{n\pm}^2(x)$ is periodic with period d . Furthermore, because $\omega_n^* \approx n\pi$ for the various gaps, the number of oscillations increases with n . Detailed considerations lead to the result that $U_{n\pm}^2(x)$ contains n oscillations over one period d .

Because $c^2(x)$ in eqn. (1) is invariant under reflection with respect to the points $x = d_1/2$ and $-d_2/2$, $U(x)$ will be symmetric or antisymmetric. This implies that $U(-x') = \pm U(x')$ where x' is the coordinate with respect to the reflection point. $U^2(x)$ will then be even with respect to $x = d_1/2$ and $-d_2/2$, and this implies that the derivatives dU^2/dx at $x = -d_2$ and $x = 0$ have opposite signs but the same absolute value. Further the derivatives at $x = -d_2$ and $x = d_1$ are equal, in agreement with the periodicity of $U^2(x)$.

4.2. Particular behaviour of U_{n-} versus U_{n+}

We now concentrate in detail on the spatial variation of U_{n-} versus that of U_{n+} . The various functions $U(x)$ will be normalized according to the condition

$$\int_{x=0}^{d} U^2(x)\, dx = 1 \tag{13}$$

In order to obtain further information on the behaviour of $U_{n\pm}(x)$, we will prove some relations derived from the basic wave equation (1) and the properties of U considered above. We start by multiplying eqn. (1) (with $U(x)$ substituted by $U_{n\pm}(x)$) with $U_{n\pm}(x)$, and by integrating over one period for $U_{n\pm}(x)$. This period is $2d$ for n odd, and d for n even. Using the Bloch property and the normalization condition (13) one arrives at (both for n even and odd)

$$\omega^2_{n\pm} = - \int_{x=0}^{d} c^2(x)\; U_{n\pm}(x)\; (d^2 U_{n\pm} / d x^2)\; dx \tag{14}$$

We want to transform this equation using partial integration. But in this respect the behaviour of $c(x)$ is important, because this function shows discontinuities. In the present paper we only consider the case of a piecewise constant function $c(x)$, with discontinuities in the points $x = x_j$. Using partial integration in the intervals where $c(x)$ is constant, the Bloch property and the continuity of $U(x)$ and dU/dx , eqn. (14) can be transformed into

$$\omega^2_{n\pm} = \frac{1}{2} \sum_j \left(\frac{d U^2_{n\pm}}{dx} \right)_j (c^2_{j+} - c^2_{j-}) + \int_{x=0}^{d} \left(\frac{d U_{n\pm}}{dx} \right)^2 c^2(x)\; dx \tag{15}$$

In the case where $c(x)$ varies between the discontinuity points, supplementary terms have to be added in the r.h.s. of this equation. The parameters c_{j+} and c_{j-} in eqn. (15) are the values of $c(x)$ at respectively $x = x_j + \varepsilon$ and $x = x_j - \varepsilon$ where ε is an infinite small number. The sum in eqn. (15) runs over the various discontinuities for $c(x)$.

In the present study we have 2 discontinuities for $c(x)$, at $x = 0$ and $- d_2$. Application of eqn. (15) leads then to

$$\omega^2_{n\pm} = \frac{1}{2} (c_1^2 - c_2^2) \left[\left(\frac{d U^2_{n\pm}}{dx} \right)_{x=0} - \left(\frac{d U^2_{n\pm}}{dx} \right)_{x=-d_2} \right] + \int_{x=0}^{d} \left(\frac{d U_{n\pm}}{dx} \right)^2 c^2(x)\, dx \tag{16}$$

As argued in § 4.1 the derivatives $dU_{n\pm}^2 / dx$ for $x = 0$ and $- d_2$ have opposite values. Therefore eqn. (16) becomes

$$\omega^2_{n\pm} = (c_1^2 - c_2^2) \left(\frac{d U^2_{n\pm}}{dx} \right)_{x=0} + \int_{x=0}^{d} \left(\frac{d U_{n\pm}}{dx} \right)^2 c^2(x)\, dx \tag{17}$$

After subtracting the 2 equations applied for the cases + and - one obtains :

$$\omega_{n+}^2 - \omega_{n-}^2 = MC + SC \tag{18}$$

Here the main contribution (MC) and secondary contribution (SC) to the positive difference $\omega_{n+}^2 - \omega_{n-}^2$ are given by

$$MC = (c_1^2 - c_2^2)\left[\left(\frac{dU_{n+}^2}{dx}\right)_{x=0} - \left(\frac{dU_{n-}^2}{dx}\right)_{x=0}\right] \tag{19a}$$

$$SC = \int_{x=0}^{d} c^2(x)\left[\left(\frac{dU_{n+}}{dx}\right)^2 - \left(\frac{dU_{n-}}{dx}\right)^2\right]dx \tag{19b}$$

One of the most striking characteristics of the functions $U_{n\pm}(x)$ follows from the previous equations. Because $c_1^2 - c_2^2 > 0$ and $\omega_{n+} > \omega_{n-}$ eqns. (17), (18) and (19a) suggest that $(dU_n^2 / dx)_{x=0}$ will be positive for $U_n = U_{n+}$ and negative for $U_n = U_{n-}$. This means that $U_{n+}^2(x)$ will increase and $U_{n-}^2(x)$ decrease when entering a region of larger $c(x)$. But this will only be true if the MC term dominates the SC term in eqn. (18). That this is true can be shown as follows. By introducing second derivatives of the functions $U_{n\pm}^2$, the MC (eqn. (19a)) can be transformed into

$$MC = \int_{x=0}^{d} c^2(x)\left[\frac{1}{2}\left(\frac{d^2(U_{n-}^2)}{dx^2} - \frac{d^2(U_{n+}^2)}{dx^2}\right)\right]dx \tag{20}$$

By comparison of eqns. (20) and (19b) we thus have to show that the value of the square bracket in the integral of eqn. (20) is larger than that in eqn. (19b). This is done as follows. The functions $U_{n\pm}(x)$ are approximately proportional to the oscillating function $\cos(\bar{k}_n x + \theta_{n\pm})$ with $\bar{k}_n$ a mean wave number. $U_{n\pm}^2(x)$ is then proportional to $1 + \cos(2\bar{k}_n x + 2\theta_{n\pm})$. When the phase difference equals $\theta_{n-} - \theta_{n+} = \pi/2$, U_{n+} and U_{n-} are orthogonal functions, leading to intensities $U_{n+}^2(x)$ and $U_{n-}^2(x)$ with opposite derivatives. When $-\pi \leq 2\theta_{n+} \leq 0$ (and hence $0 \leq 2\theta_{n-} \leq \pi$) the derivative $(dU_{n\pm}^2/dx)_{x=0}$ is positive for U_{n+}^2 and negative for U_{n-}^2. Starting with the $\cos(\bar{k}_n x + \theta_{n\pm})$ approximation for $U_{n\pm}(x)$, straightforward calculation shows that the square brackets in the integral for MC (eqn. (20)) and for SC (eqn.(19b)) are proportional respectively to $2\cos(2\bar{k}_n x + 2\theta_{n\pm})$ and $-\cos(2\bar{k}_n x + 2\theta_{n\pm})$. This means that the SC term in eqn. (18) has the opposite sign of the MC term, but its absolute value is only half the MC value. The above relations are confirmed by numerical calculations.

Because of eqn. (19a) and the previous considerations, eqn. (18) becomes

$$\omega_{n+}^2 - \omega_{n-}^2 \approx \frac{1}{2}(c_1^2 - c_2^2)\left[\left(\frac{dU_{n+}^2}{dx}\right)_{x=0} - \left(\frac{dU_{n-}^2}{dx}\right)_{x=0}\right] \tag{21a}$$

$$= (c_1^2 - c_2^2)\left(\frac{dU_{n+}^2}{dx}\right)_{x=0} \tag{21b}$$

The difference $\omega_{n+} - \omega_{n-}$ is usually small in comparison with $\omega_{n\pm} = \langle c \rangle \omega_{n\pm}^* / d \approx n\pi \langle c \rangle / d$ and therefore eqns. (21a,b) lead to the following relations between the gap width and the derivative of U^2 at the interface :

$$\omega_{n+} - \omega_{n-} \approx \frac{d\,(c_1^2 - c_2^2)}{4\pi\, n \langle c \rangle} \cdot \left[\left(\frac{dU_{n+}^2}{dx}\right)_{x=0} - \left(\frac{dU_{n-}^2}{dx}\right)_{x=0}\right] \tag{22a}$$

$$= \frac{d\,(c_1^2 - c_2^2)}{2\pi\, n \langle c \rangle} \cdot \left(\frac{dU_{n+}^2}{dx}\right)_{x=0} \tag{22b}$$

Substitution of the $\cos(\bar{k}_n x + \theta_{n\pm})$ approximation for $U_{n\pm}(x)$ in eqn. (21b) learns that $(\omega_{n+}^2 - \omega_{n-}^2) \sim (dU_{n+}^2/dx)_{x=0} \sim -\sin(2\theta_{n+})$. Thus if the gap width approximates zero, θ_{n+} equals 0 or $-\pi/2$. Simple inspection turns out that under zero gap condition, both functions $U_{n\pm}^2(x)$ show an extremum at $x = 0$, and are proportional to $\sin^2 \bar{k}_n x$ or $\cos^2 \bar{k}_n x$. Hence in that case $U_{n+}(x)$ and $U_{n-}(x)$ show an extremum (antinode), or pass through 0 (node) at $x = 0$. We will point out in § 4.4 which is the relevant case in specific circumstances.

4.3. Amplitude of U in the two regions

We finally look for the exact ratio of the amplitudes of the oscillating function U(x) in the regions I and II . In region I near the origin, U(x) can exactly be written as $U_I(x) = A_I \cos(\omega x / c_1 + \varphi_I)$ and analogous for region II. These expressions are alternatives for eqns. (5a,b). In order to find the ratio A_I / A_{II} we exploit the continuity of U(x) and its derivative at the interface between I and II . For simplicity we denote the values of U and dU/dx at the interface point x = 0 respectively as U_0 and U_0' . Then the requirements that $U_I(x)$ must have this specific function and derivative leads to the conditions $A_I \cos\varphi_I = U_0$ and $A_I \sin\varphi_I = -(c_1/\omega)\, U_0'$. Combination of these conditions leads to

$$(A_I)^2 = (U_0)^2 + (c_1/\omega)^2 (U_0')^2 \tag{23}$$

After dividing this eqn. by its analogue for region II, one obtains

$$(A_I/A_{II})^2 = \frac{(U_0)^2 + (c_1/\omega)^2 (U_0')^2}{(U_0)^2 + (c_2/\omega)^2 (U_0')^2} \tag{24a}$$

$$= \frac{1 + (c_1/\omega)^2 (U_0'/U_0)^2}{1 + (c_2/\omega)^2 (U_0'/U_0)^2} \tag{24b}$$

Because $c_1 > c_2$ eqn. (24b) tells us that

$$A_I \geq A_{II} \tag{25}$$

When for particular input parameters one has $U_0 = 0$ and $U_0' \neq 0$, the r.h.s. of eqn. (24a) becomes $(c_1/c_2)^2$. Hence we can write

$$(A_I/A_{II})_{U0=0} = c_1/c_2 \tag{26a}$$

In the complementary case $U_0 \neq 0$ and $U_0' = 0$ eqn. (24b) leads to

$$(A_I/A_{II})_{U'0=0} = 1 \tag{26b}$$

4.4. Behaviour of $U_{n\pm}(x)$ in specific cases

Using the results of the previous sections, we investigate now the features of the field for a few situations. First we examine the case where $n \geq 3$. From § 3 we know that there are at least 2 real β/α values giving zero gap. For β/α in between the $(\beta/\alpha)_{z.g.}$ values the gap width will attain a maximum and, because of eqn. (22b), the slope dU_n^2/dx on the boundary between regions I and II will attain an extremum. For $n = 3$ for instance the maximum gap will arise for $\beta/\alpha \approx 1$, situated between $(\beta/\alpha)_{z.g.} = ½$ and 2.

The previous situation changes thoroughly when β/α approaches a $(\beta/\alpha)_{z.g.}$ value. We first investigate the behaviour of $U_{n\pm}(x)$ when β/α is slightly larger than a certain $(\beta/\alpha)_{z.g.}$ value. This can be done using the following arguments. When the zero gap condition is exactly fulfilled, the derivative $(dU^2/dx)_{x=0}$ is zero (see end of § 4.2), and therefore $U_{n\pm}^2(x)$ attains an extremum at $x = 0$ (zero value or maximum). But when (for a certain α) $\beta = d_1/d_2$ becomes somewhat larger than $\beta_{z.g.}$, zone I becomes wider and the boundary between zone I and II shifts to the left. Hence U^2 will become zero or attains a maximum for small positive x. In the first case $(dU^2/dx)_{x=0}$ will be slightly negative, and (because of § 4.2) this depicts the behaviour of $U_{n-}^2(x)$. In the second case $(dU^2/dx)_{x=0}$ is slightly positive, and this behaviour describes the function $U_{n+}^2(x)$. In the first case ($U_-(x)$ shows a node for small positive x) $U_0 \approx 0$ and $U_0' \neq 0$ in the notation of § 4.3, and because of eqn. (26a) one has $A_I/A_{II} \approx c_1/c_2$. In the second case ($U_+(x)$ shows an antinode for small positive x) one has $U_0 \neq 0$ and $U_0' \approx 0$, so that eqn. (26b) results in $A_I/A_{II} \approx 1$.

The situation in which β/α is slightly smaller than a certain $(\beta/\alpha)_{z.g.}$ value, can be analysed in an analogous way. Now the boundary between zone I and II (for $x = 0$) will be situated to the right hand side of the points where $U^2(x)$ is extremum. This time the function $U_-(x)$ has an antinode for small negative x , and for this field $A_I/A_{II} \approx 1$. In contrast with this $U_+(x)$ will show a node for small negative x , while $A_I/A_{II} \approx c_1/c_2$.

We now consider the situation for $n = 1$, which deviates in several aspects from the higher n cases. For $n = 1$ the gap can only disappear if $\beta/\alpha = 0$ or ∞ . As indicated in § 4.1 the functions $U_{1\pm}^2(x)$ show only one oscillation in an interval d . Also here $A_I > A_{II}$ as shown in § 4.3. However this does not imply that $< U^2(x) >_I$ (the mean value of $U^2(x)$ in zone I) will be larger than $< U^2(x) >_{II}$. One has to distinguish between the functions $U_{1-}(x)$ and $U_{1+}(x)$ for the following reasons. From § 4.2 we know that $(dU_{1-}^2/dx)_{x=0} < 0$ and $(dU_{1+}^2/dx)_{x=0} > 0$. But because $U_{1-}^2(x)$ shows only one oscillation per interval d , this function will become zero in one point in zone I , and will attain one maximum in zone II . Hence $U_{1-}(x)$ shows one node in region I and one antinode in region II . So $< U_{1-}^2(x) >_{II}$ will be larger than $< U_{1-}^2(x) >_I$ although A_{II} is smaller than A_I . In an analogous way it can be shown that the function $U_{1+}(x)$ shows one node in region II and one antinode in I . Here, in contrast with the situation for U_{1-}, the mean value $< U_{1+}^2(x) >_I$ will be larger than $< U_{1+}^2(x) >_{II}$.

4.5. Numerical examples

Figs. 2 and 3 illustrate some of the characteristics of the field derived in the previous sections. The numerical calculations were obtained using the eqns. in § 2. In fig. 2 the fields $U_{1\pm}(x)$ and their squares are shown in the interval $-d_2 < x < d_1$ (hence $-0.4 < x^* < 0.6$) in the case $\alpha = \beta = 1.5$. Because β/α is well distinct from the zero gap values, the frequency gap is relatively large, and so are the derivatives of U^2 at the boundary between the two regions ($x = x^* = 0$). Like for arbitrary n , U_{1-}^2 decreases when entering regions of higher c (e.g. going from negative to positive x), while U_{1+}^2 increases there. Further the functions behave as described at the end of § 4.4.

In figs. 3a and 3b the fields $U_{3\pm}(x)$ and their squares are shown for $\alpha = 1.5$ and $\beta = 3.1$. For this parameters the ratio $\beta/\alpha \approx 2.07$ just surpasses the zero gap value 2.0 (see fig. 1). As argued above U_{3-} will show a node for very small and positive x , while the amplitude of the oscillations is largest in the high c regions. On the contrary U_{3+} shows an antinode for very small x , and the oscillations show the same amplitude everywhere. The fields in the case that β/α is somewhat smaller than 2.0 (not shown) behave in just the opposite way as in the previous case. Then U_{3-} behaves like U_{3+} in fig. 3b, and U_{3+} behaves like U_{3-} in fig. 3a, except that the node and antinode occur then for very small but negative x .

5. Concluding remarks

The behaviour of the standing waves $U_{n-}(x)$ and $U_{n+}(x)$ in the periodic medium could be inferred from its periodicity and parity properties (§ 4.1), and from analytical considerations starting from the basic wave equation (§ 4.2 and 4.3). This leads to eqns. (21a,b) and (22a,b), simple relations between the gap width $\omega_{n+} - \omega_{n-}$ and the derivative $(dU_n^2 / dx)_{x=0}$ at the interface between the regions of different wave velocity. Using this result it was derived that the function $U_{n+}^2(x)$ will increase and $U_{n-}^2(x)$ decrease when entering a region of larger c(x). This implies that the field $U_{n+}(x)$ will show at least one node in the low c region and one antinode in the high c region. The reverse is true for $U_{n-}(x)$ (one antinode in the low c region and one node in the high c region).

The slope of the function $U_n^{\ 2}(x)$ at the interface will largely be determined by the dimensionless parameter $\beta/\alpha = (d_1 / d_2)(c_2 / c_1)$ for the medium. When β/α is well distinct from the zero gap values $(\beta/\alpha)_{z.g.}$ (given by eqn. (11)), this slope will have a relative large absolute value. For β/α in the neighbourhood of the $(\beta/\alpha)_{z.g}$ values, detailed analysis gives the following results. When β/α is slightly larger than a $(\beta/\alpha)_{z.g}$ value, the function U_{n+} shows an antinode and U_{n-} a node near the interface, in the region of the highest c. For U_{n+} the amplitudes of the oscillations are equal in the two regions, and for U_{n-} the amplitude is the largest in the region with the highest c. (The ratio of the amplitudes is then c_1 / c_2.) When β/α is slightly smaller than some $(\beta/\alpha)_{z.g}$ the complementary situation arises. U_{n+} and U_{n-} have now respectively a node and an antinode near the interface, at the low c side. This time the amplitude of the oscillations is constant everywhere for U_{n-}, and the amplitude for U_{n+} is larger in the large c region.

The above analysis can be used in order to interpret remarkable results found by Kuzmiak et al. [11]. These authors have studied the photonic band structure in periodic systems consisting of metallic components (thin regions showing dissipation) in combination with vacuum or a dielectric. They find a remarkable asymmetric behaviour of the absorption coefficients and electromagnetic wave lifetimes for wave vectors near the Brillouin zone boundaries. This asymmetry is represented by a decreasing absorption coefficient for waves with frequencies near the lower band edge at the Brillouin zone boundary, and a significant increase for waves with frequencies in the neighbourhood of the upper band edge at the zone boundary. In terms of the above results this can be understood as follows. Because of the large electronic density the relative dielectric constant for metals is smaller than 1 (plasma model), and the metallic regions thus have the highest c value. For the low frequency standing wave the field amplitude will then decrease when entering the metallic regions. Because the latter are thin, the field will show a node in the metallic region, and little dissipation will occur, indeed leading to small absorption and a long wave lifetime. At the high frequency side, the field amplitude will increase when entering both sides of the thin metallic layers, and the field amplitude will show a local maximum. This indeed leads to a large absorption and a short wave lifetime.

The previous explanation of the asymmetric absorption behaviour can also be given in terms of the parameter β/α. For $\beta = 0$ the zero gap situation occurs (only one medium present). In the described situation β is small but not zero, so that we are in the case where β/α is slightly larger than $(\beta/\alpha)_{z.g}$. As argued above this leads to a node in the high c

region for the small frequencies (hence small absorption) and to an antinode for the high frequencies (large absorption).

The above example of the asymmetric absorption behaviour is an illustration of the influence of the characteristics of the photon or phonon field on the physical properties of a periodic layer system. It was shown that very small variations for instance of the ratio β/α can thoroughly change the spatial variation of the field and hence of the physical properties. Recently it was shown by a number of authors (see [26-29] and references given there) that it is possible indeed to vary the characteristics of a periodic medium, even in a continuous way.

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Figure captions :

Figure 1.

Dimensionless band gap width as a function of the parameter $\beta/\alpha = (d_1 / d_2) (c_1 / c_2)^{-1}$ for the 3th band gap. Zero gap width occurs at $\beta/\alpha = (\beta/\alpha)_{z.\,g.} = 0 , ½ , 2$ and ∞ .

Figure 2.

Dependence of $U_{1\,-}{}^2$ and $U_{1\,+}{}^2$ (resp. (a) and (b)) on $x^* = x / d$ for $\alpha = \beta = 1.5$. Only the interval $- d_2 < x < d_1$ $(- 0.4 < x^* < 0.6)$ is shown. The features of the curves are described in detail in § 4.4 and 4.5.

Figure 3.

Dependence of $U_{3\,-}{}^2$ and $U_{3\,+}{}^2$ (resp. (a) and (b)) on $x^* = x / d$ for $\alpha = 1.5$, $\beta = 3.1$. The β/α value just surpasses the zero gap value 2.0 . The features of the curves are described in detail in § 4.4 and 4.5.

Knuyt G. & Nesládek M. - Figures.

Fig. 1 :

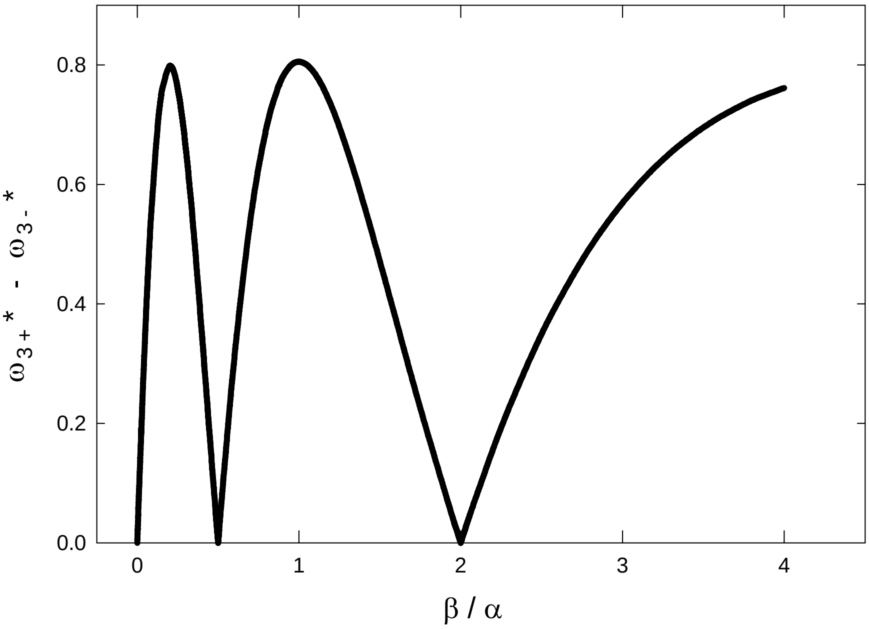
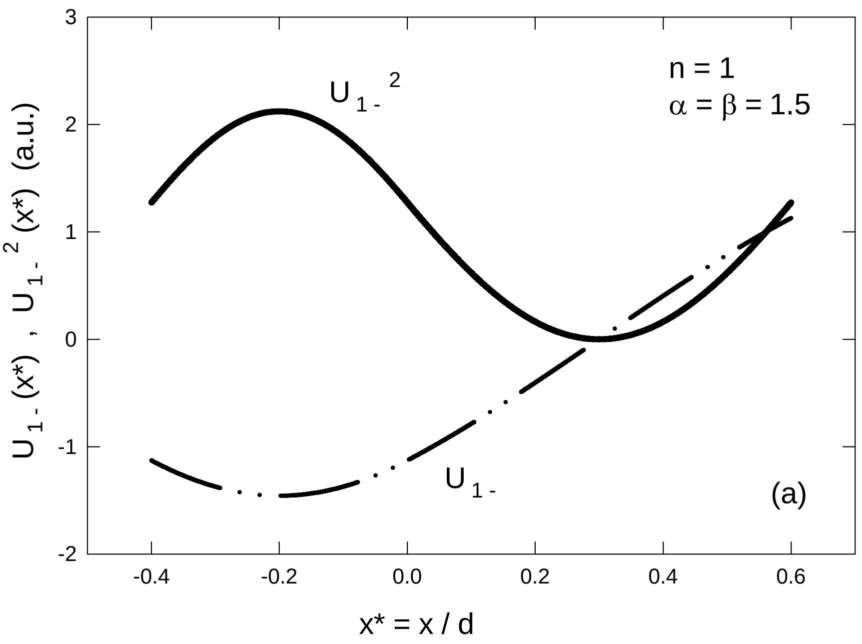
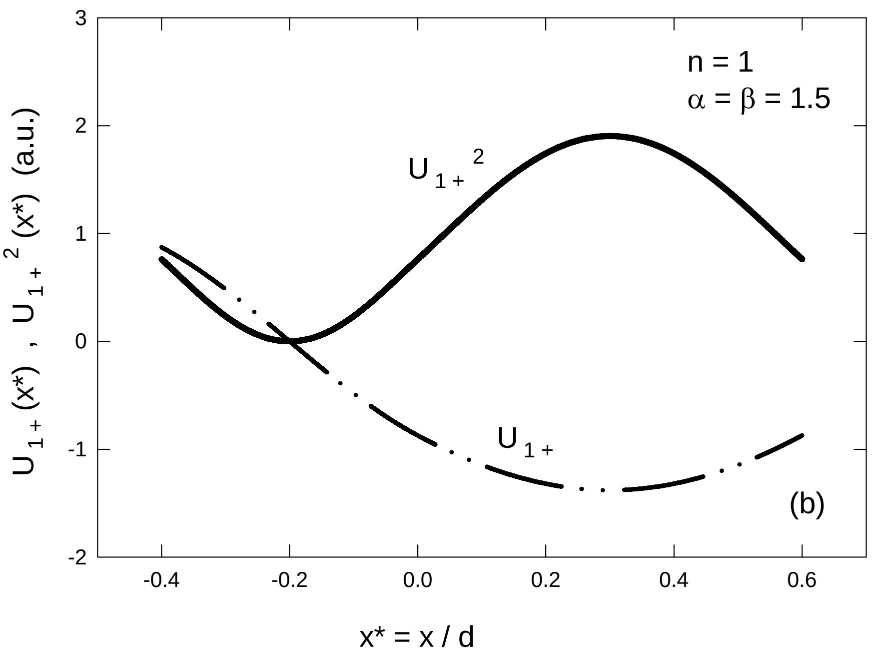


Fig. (2a) & (2b) :



3
2
1
0
-1
-2
$U_{1-}(x^*)$, $U_{1-}^{\ 2}(x^*)$ (a.u.)
$U_{1-}^{\ 2}$
U_{1-}
n = 1
α = β = 1.5
(a)
-0.4
-0.2
0.0
0.2
0.4
0.6
x* = x / d



3
2
1
0
-1
-2
$U_{1+}(x^*)$, $U_{1+}^{\ 2}(x^*)$ (a.u.)
$U_{1+}^{\ 2}$
U_{1+}
n = 1
α = β = 1.5
(b)
-0.4
-0.2
0.0
0.2
0.4
0.6
x* = x / d

Fig. (3a) & (3b) :